ON THE PARADOX OF MEDIOCRACY*

Qiang Fu†
National University of Singapore

Ming Li‡
Concordia University, CIRANO, and CIREQ

Xue Qiao§
Renmin University of China

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Abstract

We consider a two-agent hierarchical organization with a leader and a specialist in a reputation-signaling model. The specialist proposes an innovative but risky project to the leader, and decides whether to exert an effort to improve the value of the project, which benefits the organization. The leader decides whether to endorse the project or block it. The leader's competence is her private information, and the market updates its belief about the leader's type based on observation of her action (endorsing the project or blocking it) and its outcome. In equilibrium, the leader could behave excessively conservatively when she is subject to reputation concerns. We have two main findings. First, aside from its usual distortionary effects, the leader's reputation

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†Department of Strategy and Policy, National University of Singapore, 15 Kent Ridge Drive, SINGAPORE, 119245, Tel: (65)65163775, Email: bizfq@nus.edu.sg.

‡Department of Economics, Concordia University, 1455 Boulevard de Maisonneuve Ouest, Montreal, QC, H3G 1M8, CANADA, Tel: +1(514)8482424 Ext. 3922, Email: ming.li@concordia.ca.

§School of Economics, Renmin University of China, Beijing, CHINA, Tel: (86)10-82500311, Email: qiaoxue@ruc.edu.cn.

concern has a beneficial effect by inducing the specialist to supply productive effort and improves the organization's performance. Second, there exists a nonmonotonic relationship between the perceived competence of the leader and the performance of the organization. As a result, a paradox of mediocracy emerges: The organization may benefit from a seemingly mediocre leader, as a mediocre leader motivates the specialist to exert effort, which offsets the efficiency loss due to incorrect decisions.

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Everyone knew he was brilliant, but the presentation showed just how brilliant he was.

—Anonymous Bank One board member on Jamie Dimon in the bank's search for a new CEO, 1999

1 Introduction

In the selection of political, business, and professional leaders, it is often argued that preference should be given to candidates with a higher reputation for—i.e., a more favorable estimate of—competence. Such reputation for competence is often exemplified by a stellar career track record and/or superior academic credentials. This broadly epitomizes the idea of meritocracy. The pursuit of meritocracy is evidenced by the ascent of external CEO markets, the hype surrounding celebrity CEOs, and the turnover in C-suites at troubled firms. In politics, candidates' knowledge of policy and ability to learn are frequently evaluated and commented on by voters and pundits alike, and form an important basis for voters' electoral choices. Consider, for instance, the challenge posed by Howell Raines—then-executive editor of the *New York Times*—regarding George W. Bush before the 2004 US presidential election. He espoused the importance of intelligence for the presidency and asked, "Does anyone in America doubt that Kerry has a higher IQ than Bush?", which sparked a heated debate. As pointed out by Kirkpatick and Locke (1991), intelligence is one of the most sought-after traits for successful leadership: Cognitive ability "is an asset to leaders because leaders must gather, integrate, and interpret enormous amounts of information," which ensures informed decision and sensible judgment.

However, highly regarded political leaders, business executives, and policymakers often underperform relative to their reputations.[1] Management scholars have long recognized that an organization's success requires not only a prescient leader, but also involvement,

[1] Finkelstein (2004) and Malmendier and Tate (2009) present many such examples.

engagement, and the ability to motivate employees. In this paper, we urge caution regarding the conventional wisdom of meritocracy in relation to the selection of leaders. We identify a context in which a highly regarded leader may not maximize an organization's performance: She may, paradoxically, demotivate her subordinates and trigger a trade-off between decision quality and subordinates' input and contributions.

We build a simple model of hierarchical organization. A leader decides whether to endorse or block an innovative project with an uncertain outcome. She is reputation-concerned, in that her underlying competence is a hidden trait, and she cares about both the well-being of the organization and the inference about her underlying level of competence—i.e., reputation—upon observing the organization's performance. Reputation is important in the marketplace for leaders; according to one expert,[2] "Your leadership reputation is your most valuable asset." Not surprisingly, therefore, leaders sometimes choose actions that will enhance and maintain their reputation at the cost of the well-being of the organization under their governance. In this paper, we first demonstrate that a leader's higher reputation for competence does not necessarily translate into better performance of the organization. We further show that the leader's reputation concerns could contribute to the organization's performance, despite the inherent distortionary effect on the leader's choice.

Snapshot of the Model Our model involves a leader (she) and a specialist (he). The leader oversees the overall operation of an organization, e.g., a company or a government. The specialist is responsible for a specific division or unit inside the organization. The specialist identifies and develops projects and proposes one to the leader, who evaluates the project to decide whether to endorse or block it.

The essence of our model is that a project's outcome depends on two independent characteristics: *value* and *match*. The former relates to a micro-level output created by the specialist's division by executing the project, which depends on his effort. The latter, in contrast, is a macro-level output that measures how well the project fits with the organization. A mismatch inflicts costs on the whole organization. Consider, for instance, a scenario in which a public health agency (specific division) recommends a mass quarantine that restricts activities in communities in the wake of a pandemic outbreak; such a policy could effectively slow the virus's spread in the short term, but its future social and economic impact at a national level (organization) cannot be fully assessed by the public health agency.

A project's match is determined by some underlying state of the world, which is not commonly observable, while the project's value can be enhanced by the specialist's effort before being proposed to the leader. Further, the leader's ability could be *high* or *low*, which

[2] Glenn Llopis, "One Powerful Way To Control Your Leadership Reputation," April 14, 2014, *Forbes,* https://www.forbes.com/sites/glennllopis/2014/04/14/one-powerful-way-to-control-your-leadership-reputation/#156e42703931.

is privately known to the leader. A high-ability leader forms the correct inference about the project's match, while a low-ability leader cannot. Based on (1) the leader's decision regarding the project and, in the case of approval, (2) the project's eventual outcome, the market makes an inference about the leader's competence. The leader cares about both the welfare of the organization and the market's inference. The specialist's payoff derives solely from the benefit his division receives if the project is carried out. The specialist's effort improves the project's value and hence its expected return if approved. In the context of an R&D project, such effort could include deployment of resources to the precursory state of the project or application of diligence to the screening of candidate projects. Such effort, however, has no effect on its match, which, as a macro-level characteristic, goes beyond the operational boundary of the specialist's division or the scope of his discretion or expertise.

Summary of Results and Intuitions We characterize the unique plausible equilibrium of the game. In equilibrium, a high-type leader does what is best for the organization according to her signal. In contrast, a low-type leader "postures" and rejects the project with a positive probability. She does so to strike a balance between the project's material return and her potential reputation loss—even though she believes the project brings a positive expected benefit to the organization—for fear of losing reputation in case the project fails: Note that the project's quality is revealed only if it is implemented, so blocking it prevents the market from making useful inference.

We highlight two main implications of our model that stand in sharp contrast to conventional wisdom. First, we identify a *paradox of mediocracy*—that is, a leader with a middling reputation may bring better performance to an organization than one with a more stellar reputation. The key intuition rests on how the specialist's effort incentive varies with the prior of the leader's type. Note that the specialist obtains a private benefit only when the leader endorses the project. Since the high-type leader's decision is not swayed by specialist's effort, the marginal benefit of the specialist's effort derives from the increased likelihood of the low-type leader to endorse the project.

When the market perceives that the leader is more likely to be a high type, two competing effects loom large. First, the specialist expects that the leader is less likely to be swayed by his effort. Second, for a(n actual) low-type leader, she tends to respond more significantly to an increase in the project's value; in other words, his effort "persuades" the low type more effectively when she is perceived more favourably ex ante. The former effect reduces the marginal benefit of the effort, while the latter enlarges it. We show that the latter is outweighed by the former when the leader's reputation for competence is sufficiently high, which leads the marginal benefit to strictly decrease with the leader's reputation when it becomes sufficiently high.

A high-ability leader endogenously appears to be "autocratic," while her low-ability counterpart is "persuadable." As a result, a leader with a higher prior for competence may dampen the specialist's effort supply, which could offset the positive direct effect of the leader's higher likelihood of competence—namely, better decisions regarding project choice. Therefore, seemingly mediocre leadership may maximize an organization's performance.

Second, we also demonstrate a *beneficial effect of leaders' reputation concerns.* As in the usual reputation-concerns model, we predict that the low-type leader resists beneficial innovation for fear of reputation loss. However, the presence of such distortion paradoxically motivates the specialist's effort. Absent reputation concerns, the low-type leader would always endorse the project to maximize the expected outcome, and her decision would not be affected by the effort of the specialist. The specialist, therefore, would have no incentive to devote any effort to improve the project, since such effort is only valuable to himself if it will sway the leader's approval decision.

The rest of the article is organized as follows. A brief literature review follows. Section 2 presents the model setup. Section 3 analyzes the leader's and the specialist's decisions in the unique plausible equilibrium. Section 4 presents the analysis of the relationship between the leader's perceived competence and the organization's welfare. Section 5 discusses our model assumptions and possible extensions. Section 6 concludes.

Link to the Literature

In this paper, we raise questions about the merits of meritocracy in organizations. We demonstrate that a leader's reputation for competence may negatively affect the performance of the organization she leads.

In modelling a leader as a reputation-concerned decision maker who chooses whether to take risky actions, our paper belongs to an extensive literature that dates to the seminal studies of Holmström (1982, 1999). In assuming that the decision maker has private information about her own type, our paper belongs to a growing strand of literature that includes works by Canes-Wrone et al. (2001), Chung and Eső (2013), Fox and Van Weelden (2012), Fu and Li (2014), Majumdar and Mukand (2004), and Suurmond et al. (2004), where the decision maker's choice of action becomes a signaling device that could (partly) reveal her hidden information.[3]

The economics and political science literature has long recognized that agents' reputation concerns cause distortion in their behavior and compel them to take strategic action to manipulate the inference at the cost of social welfare or corporate profits. Ashworth (2012) and

[3]In contrast, in works by Biglaiser and Mezzetti (1997), Dewan and Hortala-Vallve (2012), and Hermalin (1993), neither the decision maker nor the market knows the former's type. See also Zwiebel (1995), who assumes the decision maker's action is unobservable, and Chen (2015), who compares setups in which the decision maker does or does not know her own type.

Gersen and Stephenson (2014) provide thorough reviews of the literature that considers the perverse effects of reputation concerns. As a notable counterpoint, Suurmond et al. (2004) show that reputation concerns have the beneficial effect of motivating decision makers to acquire costly and useful information. Complementary to their work, we identify another beneficial effect of reputation concerns. We do so by considering a novel model where a reputation-concerned leader and a specialist interact within an organization hierarchy. Another strand of related literature examines reputational cheap-talk games, focusing on the strategic communication between a reputation-concerned expert and a decision maker (Chen and Ishida 2015; Liu and Sanyal 2012; Morris 2001; Ottaviani and Sørensen 2001, 2006;[4] Sanyal and Sengupta 2006; Schulte and Felgenhauer 2017). However, again, these studies do not consider how an agent's reputation concerns affect another agent's effort provision in an organizational context.

Our paper is closely related to the literature on political selection. Besley (2005) investigates different approaches to selecting politicians of desirable quality (competence and honesty). Caselli and Morelli (2004) and Messner and Polborn (2004) demonstrate that low-quality (incompetent or dishonest) citizens could endogenously enter politics and run for office because of their lower opportunity costs. In a recent paper, Chen and Suen (2021) show that an opposition leader's signaling through her reform agenda could discourage moderate citizens from joining the leadership and hence leads to a more radicalized leadership group in the future. Dewan and Squintani (2017) study the selection of political leaders in a setting in which the leader makes decisions based on the advice provided by trustworthy associates, i.e., those who hold similar ideological positions. Mattozzi and Merlo (2015) examine a political party's choice of slate of candidates and highlight the tradeoff between candidates' competence and their willingness to put effort in party building. Mattozzi and Merlo Mattozzi and MerloFelgenhauer (2013) shows that a decision maker's expertise may backfire in an informational lobbying model.

Our paper is also linked to the broad literature on leadership that examines roles played by leaders in their interactions with subordinates/followers (Canes-Wrone 2006; Chen and Suen 2021; Dewan and Myatt 2007, 2008, 2012; Hermalin 1998; Komai et al. 2007; Zhou 2016). Within the literature on leadership, our paper is particularly related to two papers on how the leader induces efforts from subordinates in an organization. Rotemberg and Saloner (1993) compare between a selfish leader and an empathic one in an environment in which subordinate's (unobservable) effort adds to the value of a project and the leader determines whether to implement the project or not. Vidal and Möller (2007) study an

[4] In particular, Ottaviani and Sørensen (2001) demonstrate, in an example, that a worse outcome could result if a committee has more able experts, because herding becomes more likely. However, experts in their models are uninformed of their own types.

optimal information sharing problem between a leader and his subordinates, and show that a leader may choose not to share his more accurate information with subordinates in order to motivate them to improve organization's performance. Different from them, we emphasize the role played by the leader's reputation concerns in motivating the subordinates' effort.

Our paper also bears a link to other papers that study hierarchical organizational structures and political systems. Cheng and Li (2019) analyze how the amount of policy experimentation is affected by decentralization, when both the central policy maker and local policy makers have reputation concerns. Hirsch (2016) studies how a leader interacts with subordinates when they have different opinions. Landier et al. (2009) explore the optimal level of "dissent" (i.e., the divergence between the two agents' preferences) in a two-agent hierarchy with a decision maker and an implementer. In contrast to these authors, we focus on the effect of prior regarding a reputation-concerned decision maker's competence on a specialist's effort supply within the organization.

Our paper is related to work by Aghion and Tirole (1997) and Foarta and Sugaya (2017).[5] Similar to them, we identify an environment in which the information advantage one player holds could lower another player's effort. The source of information advantage in our paper, however, differs from theirs. In Aghion and Tirole's and Foarta and Sugaya's models, the institutional structure entitles the formal authority or overseer access to additional information. In our paper, the leader's competence level varies, and superior competence entitles the leader to additional information. Naturally, the leader's reputation concerns play an important role in our setup.

2 Model setup

We study a hierarchical organization with two agents: a leader (she) and a specialist (he). The specialist oversees the operation of a unit or division; he identifies and recommends a new project, which is innovative but has an uncertain return. The leader decides whether to endorse the project—by letting the specialist execute the project—or block it and maintain the status quo.

The project's merit is determined by two characteristics: (1) its *value*, which refers to the additional output the division produces by executing this project, denoted by $R > 0$; and (2) its *match* with the organization's overall objective, which can be good or bad. Specifically, the project, if executed, yields a net output $u = R$ to the organization when the new project is a good match, and the project is viewed as a success. In the case in which it is a bad

[5] Aghion and Tirole (1997) study the optimal authority structure problem using a standard principal-agent model, and Foarta and Sugaya (2017) study the choice between a unified and a separated regulatory structure in both a static and a dynamic model of regulatory oversight.

match, it inflicts a cost $C > R$ on the organization and yields a net output $u = R - C < 0$, and it is thus viewed as a failure. We normalize the organization's output to zero if the status quo is maintained.

We assume that the project's value, R, and the cost of a mismatch, C, are commonly known. We use random variable $\omega \in \{N, S\}$ to represent the state of the world that determines the new project's match—which can favor either the new project (N) or the status quo (S). State N occurs with a probability $p \in (0, 1)$, while state S occurs with the complementary probability $1 - p$. The distribution of ω is commonly known, while its exact realization is initially unobservable and unverifiable.

We impose the following regularity conditions in our analysis.

Assumption 1. (a) $(1 - p)C \leq R < C$; (b) $p \geq \frac{1}{2}$.

Assumption 1(a) requires that the project generate a nonnegative ex ante expected net output to the organization, but, as asserted above, a negative one when the project fails. In other words, the project is ex ante beneficial but ex post risky because it causes a loss in the event of an unfavourable state. Assumption 1(b) caps the level of uncertainty involved in the project, i.e., the probability of success p is sufficiently large. This assumption helps render well-behaved equilibria and allows us to focus on scenarios of nontrivial strategic trade-offs. Conceptually, these conditions can be interpreted as internal quality standards the organization sets for project proposals, such that only sufficiently promising projects—i.e., with nonnegative ex ante expected net output and non-excessive uncertainty—can survive prescreening and be eligible for the leader's potential approval.

2.1 Leader's competence

Presented with the project from the specialist, the leader chooses her action a, either endorsing the project ($a = a_N$) or blocking it ($a = a_S$). However, before making the decision, the leader receives a private signal, which allows her to make an inference about ω. For instance, the signal can be a research report produced by the Department of Energy to outline the fundamentals of shale oil extraction technology, or a briefing by the Department of the Treasury to assess the fiscal burden when funding major scientific initiatives. The leader's ability to properly process information and draw informative inferences defines her type (t), which can be either high (H) or low (L). Let τ^t denote the inference made by the leader from the signal. A high-type leader makes a perfect inference, with $\tau^H = \omega$. In contrast, a low-type leader's inference, τ^L, is entirely uninformative, so she maintains her prior regardless.

The leader's type, $t \in \{H, L\}$, is privately known to the leader herself. It is commonly known that the leader's initial reputation, the probability that she is of the high type, is $\pi < 1$. We assume that the specialist's information of the leader's reputation is the same as that of the public. In other words, the specialist does not have superior insight into the leader's competence.

2.2 Specialist's role

In our model, the specialist plays an active role: He can mobilize his division's resources to improve the project's value. His choice of effort is binary, $e \in \{0, 1\}$, whose influence over the value of the project is as follows:

$$R = R_0 + e\Delta,$$

where $R_0 \geq 0$ is the base value of the project, and $\Delta > 0$ is the value increment resulting from the specialist's effort. The effort e can be interpreted from diverse angles. It can be viewed as the division's early investment to prepare for its future execution. It can also be interpreted as the input committed to inventing and designing innovative initiatives. Alternatively, it can be viewed as a search or screening effort that allows the division to identify more promising ideas. The effort costs $k > 0$ to the specialist's division.

In our framework, the distinction between *value* and *match* is crucial. The former is the output at a *micro* level, i.e., the contribution through the operation of an individual division. The latter is evaluated at the *macro* level and determined by the overall interests of the organization. The specialist's effort improves the output of his own division—i.e., by improving the project's value—but does not affect the project's nature or its match with the central mission of the organization. That is, R increases with e, while p and C are independent of it.

For clarification, Assumption 1 puts restrictions on R both when $e = 0$ and when $e = 1$, which translate into the following equivalent condition:

$$(1 - p)C \leq R_0 < C - \Delta.$$

The interpretation of this condition is twofold: first, the project must meet the quality standard of the organization in order to qualify for the leader's review regardless of the specialist's effort; second, while the specialist's effort improves the project's value and its ex ante expected net output, the increment does not suffice to overcome the cost to the organization in the case of a mismatch.

It should be noted that our analysis does not explicitly require that the specialist's effort itself be observable to the leader. Nevertheless, we assume that the leader, when making the decision, learns about the project's actual value, R—which can be either R_0 or $R_0 + \Delta$. We further discuss this assumption in Section 5.

2.3 Reputation and payoffs

The market observes (1) whether the leader has allowed the specialist to execute the project, a_N or a_S, and (2) the project's actual outcome if it has been executed. The market forms its posterior about the true type of the leader—i.e., the probability of her being the high type—by Bayes' rule,

$$\lambda(u, a) = \Pr(t = H|\, u, a),$$

where u is the project's net output and a is the leader's action choice. Note that the analysis does not require that the market observe the exact output of the new project, and only requires that the qualitative outcome—i.e., success or failure—be observable.

The leader cares about both the organization's welfare and her market reputation. She has a payoff function

$$y^t(u, a) = \gamma u + (1 - \gamma)\lambda,$$

which is a weighted sum between the material net output u and her reputational payoff λ, with $\gamma \in (0, 1)$.[6]

Execution of the project yields a private benefit $D > 0$ to the specialist's division. The specialist cares only about this private benefit and the cost his division incurs from its effort to improve the project. Therefore, his payoff can be written as

$$y_m(e) = \Pr(a_N|\, R, \pi) \cdot D - ek,$$

where $\Pr(a_N|\, R, \pi)$ is the overall probability of the project's being endorsed for a given value R and a given prior π, and k is the cost incurred by the division if it exerts an effort $e = 1$.

2.4 Timeline

The timeline of the interaction is summarized as follows.

1. The specialist chooses his effort level e; he then proposes the project to the leader.

2. The leader observes the project's value R and makes an inference about its match; then, based on her inference τ^t, she decides whether to endorse the project (a_N) or block it (a_S).

3. The state of the world ω is realized, and the project, if executed, yields an output u; the public forms a posterior λ about the leader's type based on both her decision and the actual outcome of the project.

[6] In Subsection 4.2, we compare our findings with that of a model with no reputation concerns ($\gamma = 1$) and demonstrate the beneficial effect of reputation concerns.

2.5 Equilibrium concept

Note that based on the above timeline, the game consists of two stages. The first stage involves the specialist's effort choice and the second stage the leader's decision. We will therefore solve the second stage of the game first and use backward induction to solve the first stage. Given that the value of the project is observable to the leader, as long as there is a unique equilibrium outcome in the second stage, solving for the specialist's optimal effort choice is straightforward. Our discussion of the equilibrium concept is therefore focused on the second stage.

In the second stage, let a type-t leader adopt a behavioral strategy $\rho_t(R, \tau)$, which is her probability of endorsing the project for given value R and her inference τ about the match. We adopt the notion of Perfect Bayesian Equilibrium (PBE) as the solution concept for subgames with given R.

In addition to the usual requirements of PBE, we impose the following *sincerity* condition—analogous to Fu and Li (2014)—to rule out the "perverse" equilibria that typically arise in models of career and reputation concerns (Levy 2007; Prat 2005), in which the leader signals her competence by deliberately choosing a "wrong" action—i.e., the action that opposes her informative inference. Let $\rho_H(R, \tau^H)$ be the probability that the high-type leader endorses the project when the project has value R and the high-type leader makes inference τ^H.

Definition 1. An equilibrium is *sincere* if and only if $\rho_H(R, N) \geq \rho_H(R, S)$.

The sincerity condition demands that a high-type leader be more willing to endorse the project when the signal is favorable, which ensures that a leader would not be penalized when the project succeeds in terms of her reputational payoff. Note that this condition is weaker than the requirement imposed by Fu and Li (2014). As shown later, this suffices to ensure that in the equilibrium, the high type makes the best use of her superior information, although perverse signaling is not literally ruled out.

Further, we impose the popularly adopted D1 condition (Cho and Sobel 1990) to discipline out-of-equilibrium beliefs. The D1 condition demands that the out-of-equilibrium belief assign no weight to a type of agent if she is less likely to benefit from a given deviation than another type. We adapt the standard D1 criterion to our setting. A formal definition of the alternative condition is provided in the Appendix.

It is worth noting that the two requirements—i.e., sincerity and D1—complement each other. The restriction of sincerity is imposed everywhere: The leader is required and believed to behave sincerely, even when she hypothetically deviates from the equilibrium path. This nuance strengthens the D1 test in disciplining out-of-equilibrium beliefs.

3 Analysis of the leader's and the specialist's decisions

In this section, we first analyze the decision of the leader, faced with the project with known value, R. In a benchmark case, we consider the leader's decision when she has no reputation concerns. Then, we characterize the equilibrium strategy played by the leader when she does have reputation concerns. Based on this analysis, we explore the main properties of the equilibrium. Finally, we characterize the equilibrium effort decision of the specialist, anticipating what the leader will do with the project, depending on its value.

3.1 Benchmark: The leader's decision without reputation concerns

Suppose that the leader is not subject to reputation concerns—i.e., $\gamma = 1$—in which case she simply maximizes the material payoff u. The optimal decision is straightforward. The high-type leader follows her own inference: She endorses the project when her (informed) inference is favorable, and blocks it otherwise. The low type, in contrast, endorses the project with probability one, since without informative inference she maintains her prior and expects a nonnegative expected return from the project:

$$E(u) = R - (1-p)C \geq 0.$$

Note that conditional on the value of the project at the time of decision by the leader, the leader's decision is efficient and there is no distortion.

3.2 Leader's decision under reputation concerns

We now consider the leader's decision when she has reputation concerns. We first obtain an intermediate result that characterizes the high type's behavior.

Lemma 1. *In any sincere D1 equilibrium, the high-type leader endorses the project with probability one when she obtains an inference favorable to the project—i.e.,* $\rho_H(R, N) = 1$ *for* $\tau^H = N$*—and blocks it with probability one when the inference favors the status quo—i.e.,* $\rho_H(R, S) = 0$ *for* $\tau^H = S$*.*

This Lemma states that the high type's action simply follows her own inferences, as in the case without reputation concerns. Intuitively, from a high-type leader's perspective, there is no downside to rejecting a project that is a bad match, as doing so both benefits the organization and signals her competence, given the low type's higher propensity to approve a bad but ex ante beneficial project. On the other hand, when she infers that the project is a good match, she might suffer a reputation loss from approving it, as she will be pooled together with the low type. Nevertheless, the trade-off tends to tilt toward approval, given that the high type's loss is smaller than the low type's, because the former is informed about the match of the project.

The lemma allows us to focus on the low type's strategy. Because a low-type leader is unable to make informative inferences, her choice does not depend on her inference τ^L. The notation of her behavioral strategy reduces to ρ_L, her probability of endorsing the project. When the status quo is maintained, the market forms a posterior

$$\lambda^S = \frac{\pi(1-p)}{\pi(1-p) + (1-\pi)(1-\rho_L)}.$$

When the leader endorses the project and the project is successful, the market's posterior is given by

$$\lambda_s^N = \frac{\pi}{\pi + (1-\pi)\rho_L}.$$

In the case in which the project fails, the public would infer immediately that the leader is of the low type, i.e.,

$$\lambda_f^N = 0.$$

Clearly, λ^S strictly increases with ρ_L: The more often the low type endorses the project, the more likely a leader who rejects it is of the high type. In contrast, λ_s^N strictly decreases with ρ_L: The more often the low type endorses the project, the more likely a successful outcome is due to the luck of an incompetent leader rather than the sound judgment of a competent one.

If the low-type leader endorses the project, she obtains an ex ante expected material payoff $R - (1-p)C$ and an expected reputational payoff $p\lambda_s^N$. If she blocks the project, she ends up with a material payoff of zero and a reputational payoff λ^S. The equilibrium strategy of the low type is determined by the tension between material gain and reputation concerns, i.e., she endorses the project with probability one if

$$\gamma\left[R - (1-p)C\right] + (1-\gamma)p\lambda_s^N > (1-\gamma)\lambda^S,$$

or, equivalently,

$$\gamma\left[R - (1-p)C\right] > (1-\gamma)\left(\lambda^S - p\lambda_s^N\right),$$

and mixes if they are instead equal. With a slight abuse of notations, let us define

$$\tilde{u} = \frac{\gamma}{1-\gamma}\left[R - (1-p)C\right] \geq 0, \tag{1}$$

which can be interpreted as the low-type leader's valuation of the project's material return, measured vis-à-vis reputation payoffs. We define two cutoffs:

$$\begin{aligned} \bar{\lambda} &= 1 - \pi p; \\ \underline{\lambda} &= \frac{\pi(1-p)}{\pi(1-p) + (1-\pi)} - p, \end{aligned}$$

with $\underline{\lambda} < \bar{\lambda}$. We obtain the following.

Proposition 1. *For given e and therefore R, there exists a unique sincere D1 equilibrium. In the equilibrium, the high type follows her own inferences and takes her action accordingly, as stated in Lemma 1. For the low type, (i) when $\tilde{u} \leq \underline{\lambda}$, she blocks the project, i.e., $\rho_L^* = 0$; (ii) when $\tilde{u} \in \left(\underline{\lambda}, \bar{\lambda}\right)$, she randomizes between endorsing the project and maintaining the status quo. There exists a unique $\rho_L^* \in (0,1)$, which solves the equilibrium probability of her endorsing the project:*

$$\tilde{u} = \left[\frac{\pi(1-p)}{\pi(1-p)+(1-\pi)\left(1-\rho_L^*\right)}\right] - \left[\frac{\pi p}{\pi+(1-\pi)\rho_L^*}\right].$$

(iii) when $\tilde{u} \geq \bar{\lambda}$, she endorses the project, i.e., $\rho_L^* = 1$.

In the above Proposition, (iii) refers to a case in which the leader's reputation concerns are inconsequential: She chooses to pursue the project and ignore the risk of reputation loss entirely. This scenario reduces the game to the benchmark case. To restrict our attention to the most relevant cases in which reputation concerns have an impact on the leader's decision-making, we impose the following assumption in subsequent analysis.

Assumption 2. $\tilde{u} = \frac{\gamma}{1-\gamma}\left[R-(1-p)C\right] < 1-p$.

Assumption 2 means that the leader is subject to nontrivial reputation concerns (large $1-\gamma$ or small γ, ceteris paribus): Her valuation of material return measured vis-à-vis reputation payoffs—i.e., $\tilde{u}$—cannot be excessively large.

In contrast to the benchmark case without reputation concerns, sufficiently strong reputation concerns result in the usual distortion to the low-type leader's behavior: acting conservatively by blocking the project with a positive probability despite the positive ex ante expected output, which is labeled "posturing" in the reputation-concerns literature. The low-type leader exercises her discretion to block the project and keep the status quo, which functions as a safe haven to hide her incompetence, thereby leading to efficiency loss compared to the benchmark case.

If the project's ex ante return is insufficient to compensate for the reputation loss resulting from undertaking the project, or, alternatively, the leader is sufficiently reputation-concerned—i.e., when $\tilde{u}$ is sufficiently small—she would rather block the project and forgo the material gain entirely. This scenario is reflected by the condition for Proposition 1(i), which requires $\tilde{u} \leq \underline{\lambda}$. When valuation $\tilde{u}$ falls in a moderate range, i.e., in the interval $\left(\underline{\lambda}, \bar{\lambda}\right)$, the equilibrium is interior: The low-type leader balances the two sources of concern by randomizing between endorsing the project and blocking it. She randomly blocks the project to protect herself from excessive reputation loss, and also endorses it with a positive probability for the material gain; she is indifferent between them. In this case, the (relative) valuation of the project, $\tilde{u}$, exactly compensates for her reputation loss $\lambda^S - p\lambda_s^N$ when undertaking this project.

Further, recall Assumption 1(a) above. Absent it, the project would otherwise yield a negative ex ante expected net output, in which case the leader endorses the project only under rather restrictive and limited conditions.[7]

3.3 Properties of the leader's equilibrium decision

The equilibrium is determined by the tension between the low-type leader's valuation of material return, $\tilde{u}$, and her expected reputation loss from approving the project, $\lambda^S - p\lambda_s^N$. An interior equilibrium, as depicted by Proposition 1(ii), requires a balance between the two competing concerns

$$\tilde{u} = \lambda^S - p\lambda_s^N.$$

As our focus is to examine how the prior of the leader's competence affects the welfare of the organization through its effect on the specialist's effort supply, comparative statics of the effect of a higher value of the project, R, and the prior, π, will be useful in our subsequent analysis. Define $\tilde{\rho} = (1-\pi)\rho_L$, which is the overall probability of the project's being endorsed by a low type. We obtain the first comparative static property as the following.

Proposition 2. *In the interior equilibrium, the low-type leader endorses the project more often when her valuation of material return measured vis-à-vis reputation payoffs, $\tilde{u}$, increases.*[8]

This result is intuitive. Recall (1) and the equilibrium condition $\gamma\left[R-(1-p)C\right]+(1-\gamma)p\lambda_s^N = (1-\gamma)\lambda^S$. Hold other parameters fixed and let R increase, which increases $\tilde{u}$. The left-hand side increases accordingly. An increase in ρ_L is required to restore the equality, because this increases λ^S and decreases λ_s^N. That is, a higher expected net output tempts the low-type leader to embrace the innovation, since the additional material gain offsets her potential reputation loss.

The prior about the leader's type, π, plays a subtler role. Competing effects loom large when π increases, and its overall effect on equilibrium strategy ρ_L is ambiguous. Intuitively speaking, a more favorable prior on competence points toward a larger reputation loss for a low type when she fails, which would discourage her from endorsing the project. Conversely, a more favorable prior implies that the market is more likely to attribute a success to a high-type leader's superior ability, which encourages her to endorse the project. It follows

[7] Precisely, we need $\frac{\pi(1-p)}{\pi(1-p)+(1-\pi)} < p$.

[8] Given the definition of $\tilde{u}$ in (1), this Proposition implies that $\partial\rho_L^*/\partial R > 0$, $\partial\rho_L^*/\partial\gamma > 0$, and $\partial\rho_L^*/\partial p > 0$, which further implies the same signs for the corresponding comparative statics of $\tilde{\rho}$.

that the probability of the project's being endorsed by a low-type leader crucially depends on the value of the prior. We first obtain the following results:

Lemma 2. *(i) There exists a cutoff* $\underline{\pi} \in (0,1)$ *that uniquely solves*

$$\tilde{u} - \frac{\underline{\pi}^2(1-p)}{\underline{\pi} + (1-\underline{\pi})p} = 0,$$

such that $\rho_L^* \gtreqless p$ *iff* $\pi \lesseqgtr \underline{\pi}$. *(ii) There exists a cutoff* $\bar{\pi} \in (\underline{\pi}, 1)$ *that uniquely solves*

$$\tilde{u} - \left[\frac{\bar{\pi}(1-p)}{\bar{\pi}(1-p) + (1-\bar{\pi})} - p\right] = 0$$

such that $\rho_L^* > 0$ *iff* $\pi < \bar{\pi}$.

Lemma 2 defines two cutoffs with $\underline{\pi} < \bar{\pi}$. This allows us to obtain the following.

Proposition 3. *The equilibrium probability of the low-type leader's endorsing the project,* ρ_L^**, strictly decreases with* π*, the prior about the leader's competence, for* $\pi \geq \underline{\pi}$*; the overall probability of the low type's endorsing the project,* $\tilde{\rho} = (1-\pi)\rho_L^*$*, strictly decreases with* π*. That is,* $\partial\rho_L^*/\partial\pi \leq 0$ *for* $\pi \geq \underline{\pi}$ *and* $\partial\tilde{\rho}/\partial\pi < 0$ *for* $\pi \in (0,1)$.

Proposition 3 shows that the low type endorses the project less often when her prior π further improves for sufficiently large π, i.e., $\pi \geq \underline{\pi}$. That is, the low-type leader behaves more prudently or conservatively when the prior about competence improves. The sign of $\partial\rho_L^*/\partial\pi$ for π remains indeterminate for $\pi < \underline{\pi}$.[9]

Despite the indeterminacy of $\partial\rho_L^*/\partial\pi$ for small π, the overall probability of low type's endorsing the project, $\tilde{\rho} = (1-\pi)\rho_L^*$, strictly decreases with π. An increase in π affects a low-type leader's incentive to endorse the project, and also decreases the likelihood that a project is endorsed by a low-type leader, which is reflected by the discounting factor $(1-\pi)$. The leader refrains entirely from endorsing the project when the prior is sufficiently optimistic, i.e., $\pi \geq \bar{\pi}$.

3.4 Specialist's effort incentive

Going back to the first stage of the game, we now formally investigate the specialist's optimal effort choice. The specialist's payoff is given by

$$\begin{aligned} y_m(e) &= \Pr(a_N|e)D - ek \\ &= [\pi p + (1-\pi)\rho_L^*(e)]\, D - ek, \end{aligned} \tag{2}$$

[9] However, we conduct numerical exercises for a large set of parameterizations. All observations demonstrate a negative relationship between ρ_L^* and π, although it is difficult to verify the pattern analytically.

where we remind the reader that D is the private benefit to the specialist's division if the project is endorsed and k is the specialist's cost of effort, with $e \in \{0, 1\}$. Note that the probability of endorsement by the leader is influenced by the specialist's effort e through an increase in R. The specialist is willing to engage in the effort if and only if the additional benefit outweighs the cost. Observe that the specialist's decision to supply productive effort depends on the leader's reputation for competence, π, which determines the probability with which the leader endorses the project.

By our assumptions, a high-type leader's decision is made solely based on whether the project is a good match, which she observes, irrespective of the value of the project (Lemma 1). Accordingly, and as is clear from (2), the specialist's effort can only benefit him through potentially increasing a low-type leader's probability of endorsement. Thus, the additional benefit the specialist expects from his effort can be written as

$$\chi = \left\{ (1-\pi)\rho_L^*|_{e=1} - (1-\pi)\rho_L^*|_{e=0} \right\} D,$$

which is the increment in the overall probability that the low-type leader endorses the project. To facilitate our analysis, for the moment, we treat $e\Delta$ as a continuous variable that varies from 0 to Δ. The additional benefit of the specialist's effort can then be rewritten as

$$\chi = \left[\int_0^{\Delta} \frac{\partial (1-\pi)\rho_L^*}{\partial (e\Delta)} d(e\Delta) \right] D,$$

where the partial derivative $\partial \left[(1-\pi)\rho_L^*\right] / \partial(e\Delta)$ is simply $\partial\tilde{\rho}/\partial R > 0$ by Proposition 2. We are then ready to explore how the prior about the leader's competence affects χ:

$$\begin{aligned} \frac{\partial \chi}{\partial \pi} &= \left[\int_0^{\Delta} \frac{\partial^2 [(1-\pi)\rho_L^*]}{\partial(e\Delta)\partial\pi} d(e\Delta) \right] D \\ &= \left\{ \int_0^{\Delta} \left[\underbrace{-\frac{\partial \rho_L^*}{\partial(e\Delta)}}_{\text{direct effect } (-)} + \underbrace{(1-\pi)\frac{\partial^2 \rho_L^*}{\partial(e\Delta)\partial\pi}}_{\text{indirect effect } (+)} \right] d(e\Delta) \right\} D. \end{aligned}$$

Two competing forces intertwine, and the overall effect remains obscure. First, because the specialist's effort only affects the low type's incentive to endorse the project, the more likely the leader is to be a high type, the less likely his effort is to pay off. This is the direct effect of π, embodied by

$$\left[\int_0^{\Delta} -\frac{\partial \rho_L^*}{\partial(e\Delta)} d(e\Delta) \right] D = -\left(\rho_L^*|_{e=1} - \rho_L^*|_{e=0} \right) D,$$

which is negative.

Second, a rise in π affects the marginal benefit of specialist's effort, χ, by changing the way the low-type leader responds to the increment in the project's value. This is the *indirect* effect of π, embodied by

$$(1-\pi)\int_0^{\Delta}\frac{\partial\left[\frac{\partial\rho_L^*}{\partial(e\Delta)}\right]}{\partial\pi}d(e\Delta)=(1-\pi)\int_0^{\Delta}\frac{\partial^2\rho_L^*}{\partial(e\Delta)\partial\pi}d(e\Delta).$$

This indirect effect asks: When the project's value improves, does a low-type leader with a higher reputation increase ρ_L more than such a leader with a lower reputation? Or vice versa? We demonstrate in the Appendix that the cross derivative $\partial^2\rho_L^*/\left(\partial(e\Delta)\partial\pi\right)$ tends to be positive. That is, the higher her reputation, the more the low-type leader increases her probability of endorsing the project when the project's value improves. The intuition is as follows. First, as stated in Proposition 2, since a higher project value (R) compensates for a larger potential reputation loss incurred by endorsing the project, the low-type leader is motivated to endorse the project more often when R increases. Second, how more often the low-type leader will endorse the project—in response to an increase in R—depends on her prior reputation π, as this determines the magnitude of the potential reputation loss incurred. It can be shown that when π improves, the increased project value's compensation for reputation loss for accommodating the innovation, i.e., the loss for a given increase in ρ_L, tends to be worth more to a low-type leader. Simply put, the low-type leader with a lower initial reputation has less to lose when the project fails.[10] As a result, when the project's value increases, the low-type leader endowed with a higher prior reputation increases her probability of endorsing the project more than her counterpart with a lower prior reputation, as the latter is more prone to endorse the project regardless and is in less need of extrinsic stimulus.

The aforementioned result of a positive interactive effect of the low type's leader initial reputation and project value on her benefit from endorsing the project implies that the indirect effect on the leader's propensity to endorse the project likely opposes the direct effect. The following proposition spells out the relationship between $\partial(1-\pi)\rho_L^*/\partial(e\Delta)$ and the leader's reputation for competence, π, which is inherently non-monotonic.

[10]The elasticities of λ^S and $p\lambda_s^N$ with respect to ρ_L are given, respectively, by

$$\begin{aligned}\frac{\partial\lambda^S}{\partial\rho_L}\frac{\rho_L}{\lambda^S} &= \frac{\frac{1-\pi}{\pi}\rho_L/(1-p)}{1+\frac{1-\pi}{\pi}\frac{1-\rho_L}{1-p}}\text{and}\\ \frac{\partial\left[p\lambda_s^N\right]}{\partial\rho_L}\frac{\rho_L}{\left[p\lambda_s^N\right]} &= -\frac{\frac{1-\pi}{\pi}\rho_L}{1+\frac{1-\pi}{\pi}\rho_L}.\end{aligned}$$

For both, the magnitudes strictly decrease with π. This implies that when π increases, the elasticities of $\lambda^S-p\lambda_s^N$, with respect to ρ_L, tend to decline.

Proposition 4. *(i) The derivative $\partial(1-\pi)\rho_L^*/\partial(e\Delta)$ strictly increases with π, i.e.,*

$$\lim_{\pi \downarrow 0} \frac{\partial^2(1-\pi)\rho_L^*}{\partial(e\Delta)\partial\pi} > 0$$

when π is in the neighborhood of zero.

(ii) With a sufficiently large p, i.e., $p \geq \left(\sqrt{6}-1\right)/\sqrt{6} \approx 0.59$, the derivative $\partial(1-\pi)\rho_L^*/\partial(e\Delta)$ strictly decreases with π, i.e.,

$$\frac{\partial^2(1-\pi)\rho_L^*}{\partial(e\Delta)\partial\pi} < 0,$$

for $\pi > \underline{\pi}$.

Recall that $\tilde{\rho} = (1-\pi)\rho_L^*$ represents the overall probability of the project's being endorsed (by a low-type leader). Proposition 4 thus demonstrates that under plausible conditions, the marginal effect of specialist's effort on the overall probability of the project's being endorsed by a low-type leader, $\partial\tilde{\rho}/\partial(e\Delta)$, tends to increase with the leader's reputation for competence, π, for small π. For relatively large p, the derivative $\partial\tilde{\rho}/\partial(e\Delta)$, i.e., the response of the overall probability of the low type's endorsing the project, would strictly decrease with π when π is sufficiently large. Note that

$$\frac{\partial^2(1-\pi)\rho_L^*}{\partial(e\Delta)\partial\pi} = -\frac{\partial\rho_L^*}{\partial(e\Delta)} + (1-\pi)\frac{\partial^2\rho_L^*}{\partial(e\Delta)\partial\pi}.$$

It can be observed that the positive indirect effect—embodied by $\partial^2\rho_L^*/(\partial(e\Delta)\partial\pi)$—is discounted by $(1-\pi)$ and diminishes as π continues to increase. This allows us to conclude the following.

Theorem 1. *With a sufficiently large p, i.e., $p \geq (\sqrt{6}-1)/\sqrt{6} \approx 0.59$, the additional benefit of the specialist's effort, χ, strictly decreases with π for sufficiently large π, i.e., $\pi \geq \underline{\pi}$.*

Figure 1 illustrates the relationship between the additional benefit of the specialist's effort, χ, and her reputation for competence.

The additional benefit could increase with π when π remains small, but strictly decrease when π is large. Recall the cutoffs $\bar{\pi}$ and $\underline{\pi}$ defined in Lemma 2. Their specific values depend on project value R, and they both strictly increase with it, which is formally proven in the Appendix. Expressing the cutoffs as functions of the project value R, the following ensues.

Theorem 2. *Suppose $k \in (\chi(\bar{\pi}(R_0+\Delta)), \chi(\underline{\pi}(R_0+\Delta))]$. There exists a unique cutoff $\pi^* \in (\underline{\pi}(R_0+\Delta)), \bar{\pi}(R_0+\Delta))$ such that for $\pi \geq \underline{\pi}(R_0+\Delta)$, $\chi(\pi^*) \gtreqless k$ if and only if $\pi \lesseqgtr \pi^*$.*

By the result, the specialist must be willing to engage in the effort for $\pi \leq \pi^*$. He would lose his incentive, however, if π exceeds the cutoff π^*.

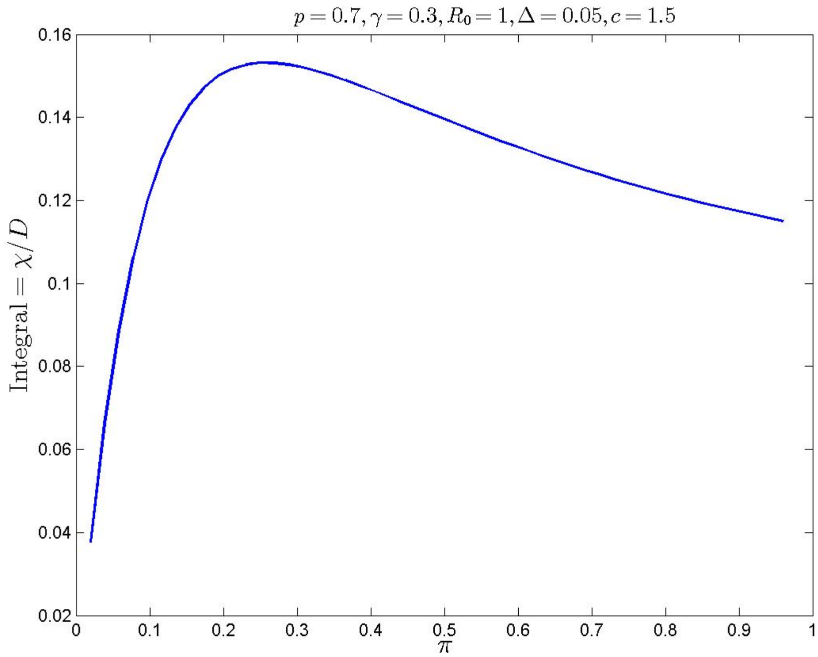

Figure 1: Additional Benefit of the specialist's Effort.

4 Welfare of the organization

In this section, we present the results of our welfare analysis.[11] First, we demonstrate that the organization may benefit from a seemingly mediocre leader. As the leader's true type is her private information, the prior π measures the leader's reputation for competence, by the public and the specialist. We show that generally, the organization's expected welfare varies nonmonotonically with the prior about the leader's type. Second, we compare the organization's welfare with its welfare without reputation concerns. We demonstrate that contrary to conventional wisdom, the presence of the leader's reputation concerns could benefit the organization, despite the distortion to her decision-making.

The organization's ex ante expected welfare can be written as a function of π:

$$\begin{aligned} u(\pi) &= \pi p R + (1-\pi)\rho_L^* \left[R - (1-p)C\right] \\ &= \pi p R + \tilde{\rho} \left[R - (1-p)C\right], \end{aligned}$$

where we have suppressed the dependence of ρ_L, $\tilde{\rho}$, and R on π. The welfare can be decomposed into contributions from the high type and the low type. A high-type leader endorses

[11] In the paper, we have abused the terminologies a bit and used "performance" and "welfare" interchangeably. This is because that our focus is the impact of the leader's reputation concern on organization's performance, rather than on the sum of the payoffs of all the members within the organization.

the project if and only if it is a good match, which occurs with probability p and yields to the organization an output R. So the high type's ex ante expected contribution is given by $\pi p R$. The low-type leader endorses the project with probability ρ_L^* and yields to the organization an expected output of $[R - (1-p)C]$. So, the low type's ex ante expected contribution is given by $\tilde{\rho}\,[R-(1-p)C]$, recalling that $\tilde{\rho} = (1-\pi)\rho_L^*$.

The specialist's effort increases R from R_0 to $R_0 + \Delta$, and benefits the organization through two venues. First, a higher R *directly* increases the organization's welfare by $\pi p + (1-\pi)\rho_L$ for a given ρ_L. Second, by Proposition 2, a higher R compels the low-type leader to endorse the project more frequently–i.e. increases ρ_L^*, which *indirectly* improves the organization's welfare: The additional material gain can offset the low type's reputation loss arising from potential failure of the project, which in turn reduces the efficiency loss caused by the low type's rejection of the (ex ante beneficial) project due to reputation concerns.

In what follows, we elaborate on this finding. In particular, it signifies that the relationship between the organization's welfare and the leader's estimated competence is, somewhat surprisingly, nonmonotonic. We interpret this as a counterargument to advocacy for meritocracy. In the meantime, we also observe that the leader's reputation concerns have a beneficial effect, which motivates her subordinate, the specialist, to devote effort to improve the project and benefit the organization.

4.1 Paradox of mediocracy: Relationship between welfare and the leader's reputation

We now formally explore how the organization's ex ante expected welfare varies with its leader's reputation for competence, based on our equilibrium characterization and associated comparative statics in the previous section. As Theorem 2 states, when the leader's reputation π exceeds a certain threshold, π^*, the specialist loses his incentive to make the effort. A paradox of mediocracy thus emerges: The organization's welfare could suffer when its leader is expected to be more competent. Figure 2 demonstrates the paradox.

In the figure, the curve on the top depicts the organization's welfare for specialist's effort being $e = 1$ and the curve at the bottom demonstrates that for $e = 0$. We then highlight in red and blue, respectively, how the organization's *actual* expected welfare varies with π under two scenarios in which effort cost, k, differs. In both scenarios, welfare drops discretely when π reaches the cutoff π^*, then continues to rise along the curve for $e = 0$. The red (kinked) curve corresponds to the scenario with a high effort cost k in which the discrete drop in the welfare occurs at a smaller cutoff π^*. In contrast, the blue (kinked) curve corresponds to the scenario with a lower effort cost k in which the discrete drop occurs at a larger cutoff π^*. Let $u(\pi|\, e)$ be the organization's welfare when its leader has a prior of π and the specialist

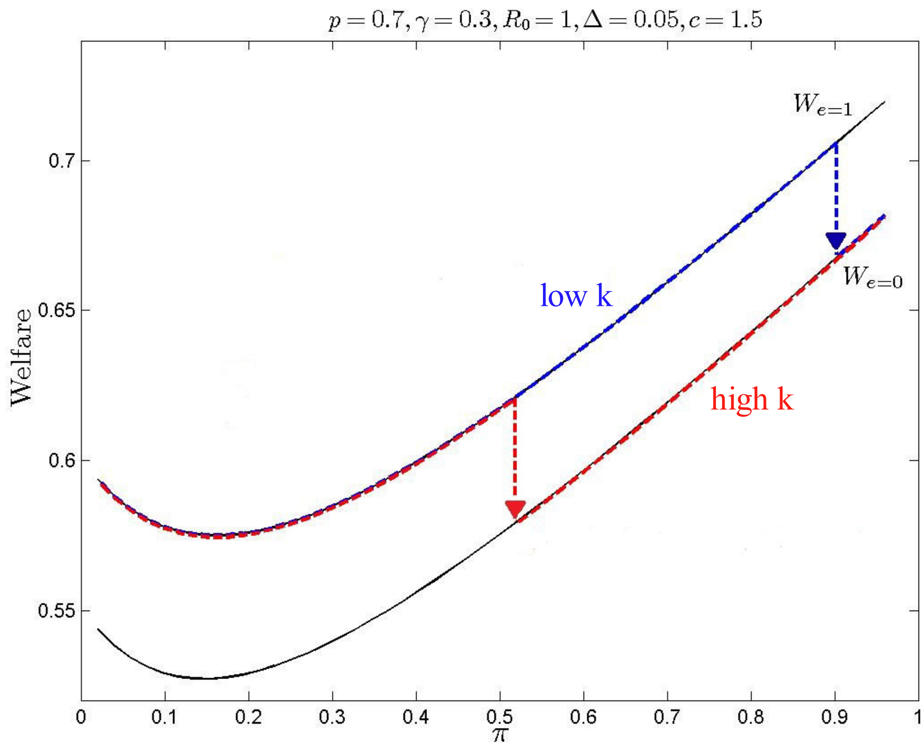

Figure 2: Welfare of the Organization and the leader's Reputation.

exerts an effort e. Given the discrete drop at π^*, it is intuitive to conclude the following.

Either π^* or 1 can be the maximizer of the organization's welfare. In the scenario depicted by the red curve (high cost of effort, k), $u(1|\,0)$ is higher than $u(\pi^*|\,1)$ despite the discrete drop at π^*. In contrast, in the scenario depicted by the blue curve (low cost of effort, k), $u(\pi^*|\,1)$ dominates $u(1|\,0)$: The rise along the curve for $e = 0$ does not offset the discrete drop at π^*.

Suppose that the organization must pick a leader from a pool, and that candidates differ in their initial reputations. The organization may prefer a seemingly mediocre leader, as demonstrated in Figure 2. One with a prior $\pi = \pi^*$—who incentivizes efforts—can paradoxically outperform even a truly competent candidate ($\pi = 1$).

Proposition 5. (Paradox of Mediocracy) *The organization's welfare is maximized at π^* when $u(\pi^*|1) > u(1|0)$. That is, the organization may benefit from a seemingly mediocre leader.*

It should be noted that the underlying trade-off remains relevant even if we assume that the specialist's effort is a continuous choice variable. In that case, his optimal effort would continuously decrease with π when π is sufficiently large. The discrete drop in expected welfare, as depicted in Figure 2 above, would not occur. Instead, we expect to observe a smooth envelope that connects the curves for $e = 1$ and $e = 0$. The main prediction remains valid qualitatively.

4.2 Beneficial effect of reputation concerns

It is worth noting that our setting also implies a beneficial effect of reputation concerns. The conventional wisdom in the literature holds that reputation concerns distort decision-making and cause inefficiency. Suurmond et al. (2004) show that a reputation-concerned decision maker can be incentivized to exert costly effort to acquire information in order to prove her own competence, which gives rise to a beneficial effect of reputation concerns.

Complementary to Suurmond et al. (2004), our paper also reveals a bright side of reputation concerns. Recall in the benchmark case that without reputation concerns, the low-type leader endorses the project with probability one, which maximizes the expected welfare of the organization. Despite the lack of distortion to the leader's decision-making, the specialist's incentive to devote effort vanishes entirely, because it does not affect the leader's decision. Without reputation concerns, the organization's expected welfare is

$$u^b = \pi p R_0 + (1-\pi)\left[R_0 - (1-p)C\right].$$

With reputation concerns in place, the expected welfare is given by

$$u = \pi p R + (1-\pi)\rho_L \left[R - (1-p)C\right].$$

where R can be either R_0 or $R_0 + \Delta$, depending on whether the specialist has sufficient incentive to make the effort. A trade-off thus arises between distortion in decision making and effort incentive. When $\pi \leq \pi^*$, the extra incentive generates a benefit

$$\left[\pi p + (1-\pi)\rho_L\right]\Delta,$$

while the distortion leads to a loss

$$(1-\pi)(1-\rho_L)\left[R_0 - (1-p)C\right].$$

The comparison depends on complex interactions among many factors, and the organization could end up with a higher return when its leader is subject to reputation concerns. Figure 3 demonstrates one such possibility.

The black dashed line depicts the maximum welfare that could result without reputation concerns, i.e., when the leader is a high type with probability one ($\pi = 1$); the blue dashed curve plots the welfare in our model, i.e., when the leader is subject to reputation concerns and the specialist chooses his efforts strategically. Without reputation concerns, the effort

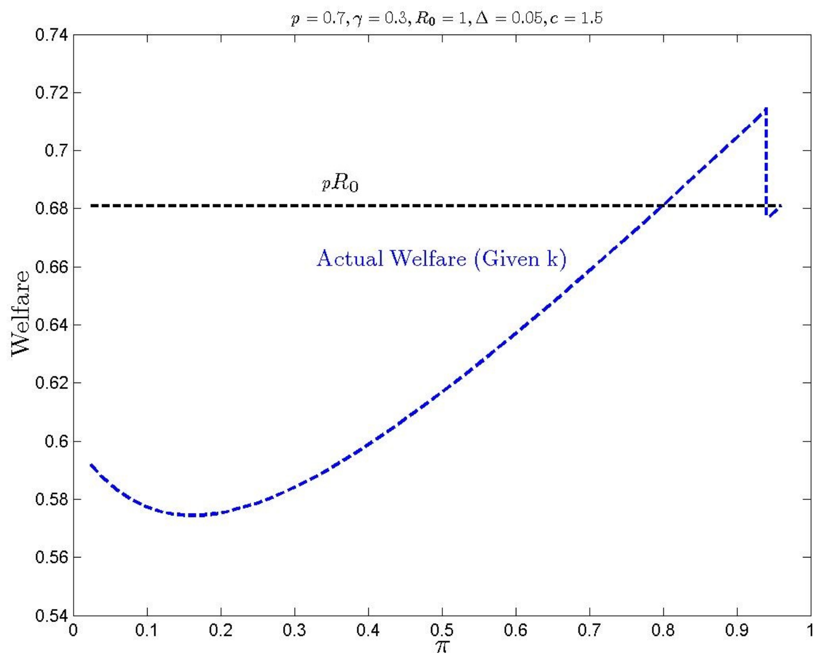

Figure 3: Beneficial Effect of Reputation Concerns.

incentive disappears, and the organization ends up with a welfare of pR_0. In contrast, in the presence of a reputation-concerned leader, the specialist is willing to engage in costly effort for $\pi \leq \pi^*$. As the figure shows, the resultant welfare could exceed pR_0, i.e., the welfare when the leader is immune to reputation concerns and even when she is truly competent.

5 Discussion

In this section, we discuss several assumptions of our model and the ramifications of alternative setups.

5.1 Drastic improvement by the specialist's effort

Recall that we assume $(1-p)C \leq R < C$ regardless of the specialist's effort, or equivalently, $(1-p)C \leq R_0 < C - \Delta$, which implies that ex ante, the project brings a positive benefit to the organization—but conditional on its being a bad match, it brings a net loss regardless of the specialist's value-improving effort.

In this subsection, we consider an alternative setting in which the specialist's effort results in a drastic improvement in value: The project yields an ex ante positive expected return if and only if the specialist puts in the required effort.[12] That is, we assume $R_0 < (1-p)C$ and $(1-p)C - R_0 < \Delta < C - R_0$. We now demonstrate that the counterproductive effect of an ex ante more competent leader may persist.

We consider a simpler case in which the leader's type is publicly known and reputation concerns are thus abstracted away. Thus a leader approves the projects if and only if she anticipates a positive expected return. A high-type leader behaves in the same manner as in our main model: She approves it in state N and blocks it in state S, because she receives a perfectly informative signal. In contrast, a low type, in this setting, would approve the project if and only if the specialist makes the effort.

Now we are ready to consider the specialist's effort decision. Suppose that a high type is in office. An improved project would not sway the leader's decision in state S and, as in the main model, the specialist will not make any effort. The organization ends up with an ex ante expected welfare of pR_0. Suppose that a low type is in office. The leader approves the project if and only if the specialist makes the effort, which incentivizes him to do so. The organization's expected welfare thus becomes $(R_0 + \Delta) - (1-p)C$. The organization benefits from a low-type leader if and only if $\Delta > (1-p)(C - R_0)$. That is, a low-type leader motivates the specialist and outperforms a high type when the effort creates a large improvement in the value of the project (but not large enough to overwhelm a mismatch).

5.2 Specialist's benefit/cost

We follow the standard moral hazard model (see Harris and Holmström 1982; Lazear and Rosen 1981) and assume that the organization's welfare does not factor in the specialist's benefit and cost.

Suppose instead that the organization takes them into account. Its ex ante expected welfare $u(\pi|\, e)$ will be rewritten as

$$\begin{aligned} u(\pi|\,0) &= \pi p(R_0 + D) + \tilde{\rho}\left\{[R_0 - (1-p)C] + D\right\}, \text{ and} \\ u(\pi|\,1) &= \pi p[(R_0 + \Delta) + (D - k)] + \tilde{\rho}\left\{[(R_0 + \Delta) - (1-p)C] + (D - k)\right\}. \end{aligned}$$

Recall Figure 2, which plots the organization's welfare in relation to the prior π. Regardless of the specialist's effort, $u(\pi|\, e)$ will be shifted upward, with $u(\pi|\, 0)$ to rise more because the effort cost is not sunk in this case. The cutoff π^* remains the same because the specialist's effort is unaffected.

Recall by Proposition 5 that the organization's welfare is maximized at $\pi = \pi^*$ if $u(\pi^*|\, 1) \geq u(1|\, 0)$ and $\pi = 1$ otherwise. It is straightforward to conclude that ex ante mediocracy is less likely to emerge in the optimum because $u(1|\, 0)$ rises more than $u(\pi^*|\, 1)$.

[12] We thank an anonymous referee for suggesting this alternative setup.

We further consider a welfare criterion in which only the effort k is taken into account. For instance, one could interpret that D accrues to the private benefit of the specialist only, while the effort involves the costly operation of the specialist's division. Then the organization's expected welfare is rewritten as

$$\begin{aligned} u(\pi|\,0) &= \pi p R_0 + \tilde{\rho}\,[R_0 - (1-p)C]\,, \text{ and} \\ u(\pi|\,1) &= \pi p[(R_0+\Delta) - k] + \tilde{\rho}\,\{[(R_0+\Delta) - (1-p)C] - k\}\,. \end{aligned}$$

Similarly, we observe that an ex ante mediocre leader is less likely to be optimal because $u(1|\,0)$ remains the same, while $u(\pi|\,1)$ is shifted down.

In conclusion, when the organization factors in the specialist's benefit and/or cost, optimal mediocracy is less likely: Optimal mediocracy arises when an ex ante mediocre leader incentivizes the specialist to improve the project, but the gain from that is diminished when the organization has to shoulder the cost.

5.3 The Observability of project value and alternative measure of the leader's competence

As stated earlier, our model does not necessarily require that the specialist's effort e be observable to the leader. However, our analysis does rely on the critical assumption that the leader can precisely assess the improvement in the project's value R. This has the implication that the low type is "persuadable" to be more accommodating in response to an improved project, which paves the way for our counterintuitive prediction.

To see the importance of this assumption, let us consider an alternative setting in which the improvement in the project value cannot be readily observed and an assessment requires the leader's expertise; that is, only the high type can observe the realization of Δ. Suppose that we maintain the parametric setting of the main model, i.e., $\Delta < C - R_0$. The high type's decision is not affected by the variation in project value. However, the low type could not effectively respond to the specialist's effort to improve the project because she cannot perceive the improvement: If she anticipated the specialist's effort to improve the project and approved it, the specialist would deviate to no effort and still be able to induce approval. We then expect the specialist to not put in any effort, and the model ends up the same as the case in which the improvement is observable to neither type.

Suppose, alternatively, that a substantial improvement $\Delta > C - R_0$ is achieved by the specialist. The model's spirit will be reversed. In this case, the high type will endorse the project even if an unfavorable state is realized. As a result, the specialist tends to be encouraged to invest when the leader is more likely to be a high type, which could overturn the incentive effect embedded in our main model. To the extent that the prior is excessively pessimistic—i.e., the proportion of low type is too high—the specialist can be entirely discouraged from making effort.

6 Concluding remarks

In this paper, we consider the interaction inside an organizational hierarchy between a reputation-concerned leader and her subordinate, a specialist. The specialist proposes a project to the leader for approval. A competent leader knows which projects are more likely to succeed. The leader's reputation concerns distort her decision-making and lead an incompetent leader to posture and resist ex ante beneficial innovation. The distortion, however, provides an incentive for the specialist to supply productive effort in order to influence the leader's decision. We show that there exists a trade-off between better decision-making by a more competent leader and provision of incentive for the specialist to devote effort to improve the project. We find that the organization may paradoxically benefit from a seemingly mediocre leader, and provide a cautionary tale for the unconditional pursuit of meritocracy in leadership positions.

Our paper leaves room for future extensions. In our model, we do not explicitly consider outcome-based contracts that motivate the specialist's effort, i.e., letting the specialist's pay depend on the actual return of the executed project. Our analysis applies more directly to a non-contractible environment—e.g., when performance pay is not feasible, as in the public sector, or when the outcome is unverifiable. The optimal design of an incentive contract warrants serious research interest. However, it should be noted that the central trade-off, i.e., the tension between the leader's estimated competence and the specialist's incentive, would remain intact. The specialist continues to expect a higher return on his effort when the leader turns out to be of low type, in which case he expects a higher probability of the project's execution, in addition to the extra pay that will result from the higher project value. The specialist may still be motivated to exert effort when he expects the leader is more likely to be of low type.

In our model, the specialist passively presents an available project. An interesting extension would be to let the specialist decide whether to propose a new project when its availability is unobservable to the market. When proposing the project, the specialist forces a leader to bear the risk of reputation loss, which allows the market to infer the leader's type and affects her chance of keeping the office. Imagine that the specialist's position is awarded as the leader's patronage, and therefore the specialist is concerned with the likelihood that the leader will be able to keep her office. The specialist's decision to propose the project must be based on the prior about the leader's type and the cycle of the leader's tenure, which catalyzes another channel through which the prior affects the organization's performance.

Finally, in our model, the prior about the leader's competence is exogenously given. Our analysis implies that a leader with a higher reputation may not always benefit the organization. On the other hand, as we demonstrate, without uncertainty about the leader's competence, the specialist would have no incentive to make an effort. Thus our results can also be viewed as an argument for keeping an appropriate level of uncertainty about the leader's competence. For a more complete study of the evolution of the reputation of a leader's competence, one may want to construct a more general dynamic model. Having a higher reputation for competence enhances the leader's market prospects and societal recognition, but may also lead to underperformance at her present organization. A fully specified model that allows entry and exit by leaders is likely to better capture the dynamics of leaders' reputation.

Appendix A. Definition of D1 Criterion

The D1 condition (Cho and Sobel 1990) imposes additional restrictions on the market's out-of-equilibrium belief. Upon observing an unexpected action (and outcome), the market infers the type of the leader who has committed the deviation. The condition demands that the out-of-equilibrium belief assigns no weight to the type of agent who is less likely to benefit from the given deviation than another type.

We hereby use an alternative definition of "type" in our context, which allows us to specify the general condition for divinity in our context. A player is of a type $\tilde{t}$, with

$$\tilde{t} = \begin{cases} (H, \tau), & \text{if } t = H; \\ L, & \text{if } t = L, \end{cases}$$

where $\tau \in \{N, S\}$ is the inference a high-type player receives before taking her action. The alternative "type" indicated by $\tilde{t}$ is defined to reflect that a high-type leader obtains informative inference and could take responsive action accordingly. For a low-type leader, the inference is noisy, so it does not factor in her action choice and a differentiation is unnecessary.

Let the market form a system of beliefs $\phi \equiv \{\epsilon_{\tilde{t}}\}$ upon observing a deviation, where $\epsilon_{\tilde{t}}$ is the probability that a type-$\tilde{t}$ leader is believed to have committed the deviation on the out–of-equilibrium path. Let $y^{\tilde{t}}$ denote the payoff a type-$\tilde{t}$ leader would receive by committing to the given deviation given the system of belief ϕ, and $y^{\tilde{t}*}$ the payoff that she would receive in the equilibrium. Further define the set $\Phi^{\tilde{t}} \equiv \{\phi |\, y^{\tilde{t}} > y^{\tilde{t}*}\}$. We then have the following.

Definition 2. Under D1 Criterion, the out-of-equilibrium belief ϕ satisfies:

$\epsilon_{\tilde{t}} = 0$ if $\Phi_{\rho}^{\tilde{t}} \subset \Phi_{\rho}^{\tilde{t}'}$, with $\tilde{t} \in \{(H, N), (H, S), L\}$, and $\tilde{t} \neq \tilde{t}'$.

Appendix B. Proofs

6.1 Proof of Lemma 1

Proof. By the sincerity requirement, $\rho_H(R, N) \geq \rho_H(R, S)$ for the high type. For the low type, the probability of endorsing the project is independent of the state or her own inference, as she maintains her prior regardless. For given R, we simplify the leader's probability of endorsing the project as $\rho_H(\tau)$ for the high-type or ρ_L for the low type.

Suppose that the project is endorsed and succeed, the leader's reputation payoff can be written as

$$\begin{aligned}\lambda_s^N &= \frac{\pi p \rho_H(N)}{\pi p \rho_H(N) + (1-\pi) p \rho_L} \\ &= \frac{\pi \rho_H(N)}{\pi \rho_H(N) + (1-\pi)\rho_L}.\end{aligned}$$

If the project is endorsed and fails, the reputation ends up as

$$\lambda_f^N = \frac{\pi \rho_H(S)}{\pi \rho_H(S) + (1-\pi)\rho_L}.$$

If the project is blocked and the status quo is maintained, the leader has a reputational payoff

$$\lambda^S = \frac{\pi \left\{p[1-\rho_H(N)] + (1-p)[1-\rho_H(S)]\right\}}{\pi \left\{p[1-\rho_H(N)] + (1-p)[1-\rho_H(S)]\right\} + (1-\pi)(1-\rho_L)}.$$

For the high type, if she chooses to block the project, she has a payoff $(1-\gamma)\lambda^S$. When $\omega = N$, if she chooses to endorse the project, she has a payoff

$$\gamma R + (1-\gamma)\lambda_s^N;$$

when $\omega = S$, she has a payoff

$$\gamma(R - C) + (1-\gamma)\lambda_f^N.$$

For the low type, the trade-off is state-independent. She simply compare

$$\gamma\left[R - (1-p)C\right] + (1-\gamma)\left[p\lambda_s^N + (1-p)\lambda_f^N\right],$$

the expected payoff for endorsing the project, with $(1-\gamma)\lambda^S$, the payoff for blocking it.

By sincerity condition, $\rho_H(N) \geq \rho_H(S)$, which ensures $\lambda_s^N \geq \lambda_f^N$, and therefore $\gamma R + (1-\gamma)\lambda_s^N > \gamma\left[R - (1-p)C\right] + (1-\gamma)\left[p\lambda_s^N + (1-p)\lambda_f^N\right] > \gamma(R-C) + (1-\gamma)\lambda_f^N$. This implies $\rho_H(N) \geq \rho_L \geq \rho_H(S)$.

Suppose equalities hold simultaneously. There are two possibilities. First, consider a hypothetical equilibrium $\rho_H(N) = \rho_L = \rho_H(S) = 1$. Imagine that the project is unexpectedly blocked. We now use the alternative definition of type $\tilde{t}$, and recall that $\epsilon_{\tilde{t}}$ is the probability that a type-$\tilde{t}$ player blocks the project. Then λ^S can be written as

$$\tilde{\lambda}^S = \frac{\pi\left[p\epsilon_{(H,N)} + (1-p)\epsilon_{(H,S)}\right]}{\pi\left[p\epsilon_{(H,N)} + (1-p)\epsilon_{(H,S)}\right] + (1-\pi)\epsilon_L}.$$

To have an incentive to deviate, the leader must receive a higher payoff than she would in the equilibrium. For $\tilde{t} = L$, this occur if $(1-\gamma)\tilde{\lambda}^S \geq \gamma\left[R-(1-p)C\right]+(1-\gamma)\left[p\lambda_s^N + (1-p)\lambda_f^N\right]$. Then it must be true that $(1-\gamma)\tilde{\lambda}^S > \gamma(R-C)+(1-\gamma)\lambda_f^N$. Similarly, for $\tilde{t} = (H,N)$, this occurs if $\gamma R + (1-\gamma)\lambda_s^N \leq (1-\gamma)\tilde{\lambda}^S$, which then implies $(1-\gamma)\tilde{\lambda}^S > \gamma\left[R-(1-p)C\right] + (1-\gamma)\left[p\lambda_s^N + (1-p)\lambda_f^N\right]$. We then obtain $\epsilon_{(H,N)} = \epsilon_L = 0$. This implies $\tilde{\lambda}^S = 1$ given the requirement of $D1$. As a result, for $\tilde{t} = (H,S)$, the equilibrium payoff must be strictly less than the payoff for the deviation, because $R - C < 0$. That is, the high type must deviate if she has an inference $\tau = S$. The equilibrium breaks down.

Second, consider the possibility of $\rho_H(N) = \rho_L = \rho_H(S) = 0$, in which case $\lambda^S = \pi$. Suppose that there is an unexpectedly endorsed project. If the project succeeds, then we have

$$\tilde{\lambda}_s^N = \frac{\pi\epsilon_{(H,N)}}{\pi\epsilon_{(H,N)} + (1-\pi)\epsilon_L};$$

if the project fails,

$$\tilde{\lambda}_f^N = \frac{\pi\epsilon_{(H,S)}}{\pi\epsilon_{(H,S)} + (1-\pi)\epsilon_L}.$$

By sincerity condition, $\tilde{\lambda}_s^N \geq \tilde{\lambda}_f^N$. Hence, if $(1-\gamma)\lambda^S \leq \gamma\left[R-(1-p)C\right]+(1-\gamma)\left[p\tilde{\lambda}_s^N + (1-p)\tilde{\lambda}_f^N\right]$, it must be true that $(1-\gamma)\lambda^S < \gamma R + (1-\gamma)\tilde{\lambda}_s^N$. This implies the out-of-equilibrium belief must specify $\epsilon_L = 0$, which leads to $\tilde{\lambda}_s^N = 1$. Then the high type must deviate when she has an inference $\tau = N$, which dissolves the equilibrium.

Hence, we conclude that $\rho_H(N) = \rho_L = \rho_H(S)$ cannot occur in any sensible equilibrium, which, in turn, implies $\rho_H(N) > \rho_H(S)$.

We claim $\rho_H(S) = 0$. Suppose otherwise that $\rho_H(S) \in (0,1)$. Then we must have $\rho_H(N) = \rho_L = 1$, which implies $\lambda^S = 1$. Hence,

$$\gamma(R-C) + (1-\gamma)\lambda_f^N < (1-\gamma)\lambda^S = (1-\gamma),$$

which leads to contradiction.

We then claim $\rho_H(N) = 1$. Suppose otherwise $\rho_H(N) \in (0,1)$. Then we must have $\rho_L = 0$, because $\gamma R + (1-\gamma)\lambda_s^N > \gamma\left[R-(1-p)C\right] + (1-\gamma)\left[p\lambda_s^N + (1-p)\lambda_f^N\right]$. In this case, $\lambda_s^N = 1$, which implies

$$\gamma R + (1-\gamma)\lambda_s^N = \gamma R + (1-\gamma) > (1-\gamma)\lambda^S,$$

which leads to contradiction.

The claim of Lemma 1 is thus verified. ∎

6.2 Proof of Proposition 1

Proof. We focus on the strategy played by the low type. If she endorses the project, the payoff amounts to

$$\gamma[R-(1-p)C]+(1-\gamma)\left[\frac{\pi p}{\pi+(1-\pi)\rho_L}\right].$$

Note that it strictly decreases with ρ_L.

If she blocks the proposal, her payoff is simply $(1-\gamma)\left[\frac{\pi(1-p)}{\pi(1-p)+(1-\pi)-(1-\pi)\rho_L}\right]$. It strictly increases with ρ_L.

Suppose $\rho_L=0$. By endorsing the project, the low type leader receives a payoff

$$\gamma[R-(1-p)C]+(1-\gamma)p.$$

By blocking it, she has a payoff

$$(1-\gamma)\left[\frac{\pi(1-p)}{\pi(1-p)+(1-\pi)}\right].$$

If $\gamma[R-(1-p)C]+(1-\gamma)p\leq(1-\gamma)\left[\frac{\pi(1-p)}{\pi(1-p)+(1-\pi)}\right]$, the leader strictly prefers blocking it to endorsing it with positive probability. In this case,

$$\gamma\left[R-(1-p)C\right]+(1-\gamma)\left[\frac{\pi p}{\pi+(1-\pi)\rho_L}\right]<(1-\gamma)\left[\frac{\pi(1-p)}{\pi(1-p)+(1-\pi)-(1-\pi)\rho_L}\right]$$

holds for any $\rho_L\in(0,1]$.

Suppose $\rho_L=1$. By endorsing the project, the lower-type leader, receives a payoff

$$\gamma\left[R-(1-p)C\right]+(1-\gamma)\pi p.$$

By blocking it, she has a payoff $(1-\gamma)$.

If $\gamma\left[R-(1-p)C\right]+(1-\gamma)\pi p\geq(1-\gamma)$, she strictly endorsing the project to blocking it. In this case,

$$\gamma\left[R-(1-p)C\right]+(1-\gamma)\left[\frac{\pi p}{\pi+(1-\pi)\rho_L}\right]>(1-\gamma)\left[\frac{\pi(1-p)}{\pi(1-p)+(1-\pi)-(1-\pi)\rho_L}\right]$$

holds for any $\rho_L\in[0,1)$.

When neither of the above two conditions holds, there must exist a unique $\rho_L^*\in(0,1)$ such that the low type is indifferent between endorsing the project and blocking it, i.e.,

$$\gamma\left[R-(1-p)C\right]+(1-\gamma)\left[\frac{\pi p}{\pi+(1-\pi)\rho_L^*}\right]=(1-\gamma)\left[\frac{\pi(1-p)}{\pi(1-p)+(1-\pi)-(1-\pi)\rho_L^*}\right],$$

by the monotonicity of $\frac{\pi p}{\pi+(1-\pi)\rho_L}$ and $\frac{\pi(1-p)}{\pi(1-p)+(1-\pi)-(1-\pi)\rho_L}$ in ρ_L. An interior equilibrium thus emerges. ∎

6.3 Proof of Proposition 2

Proof. Consider the equilibrium condition

$$\gamma\left[R-(1-p)C\right]+(1-\gamma)p\lambda_s^N=(1-\gamma)\lambda^S.$$

Recall that

$$\lambda^S=\frac{\pi(1-p)}{\pi(1-p)+(1-\pi)(1-\rho_L)}.$$

When the leader endorses the project and the project is successful, the market's posterior is given by

$$\lambda_s^N=\frac{\pi}{\pi+(1-\pi)\rho_L}.$$

Note that λ^S, reputation from keeping the status quo, is increasing in ρ_L and that λ_s^N, reputation from successful implementation of the new project, is decreasing in ρ_L. Rewriting the equilibrium condition gives

$$\tilde{u}\equiv\frac{\gamma}{1-\gamma}\left[R-(1-p)C\right]=\lambda^S-p\lambda_s^N.$$

When γ increases, R increases, or C decreases, the LHS of the equilibrium condition increases, thus ρ_L must decrease to restore the equality, as the RHS is decreasing in ρ_L. When p increases, it causes the LHS to increase and the RHS to decrease, again requiring ρ_L to decrease to restore the equality. Hence, ρ_L is decreasing in γ, R, and p and increasing in C, which also means that $\tilde{\rho}=\pi+(1-\pi)\rho_L$ is decreasing in γ, R, and p, and increasing in C. ∎

6.4 Proof of Lemma 2

Proof. (i) Recall that the equilibrium condition for ρ_L^* is

$$F(\rho_L^*,\pi)=\tilde{u}-\left[\frac{\pi(1-p)}{\pi(1-p)+(1-\pi)-(1-\pi)\rho_L^*}-\frac{\pi p}{\pi+(1-\pi)\rho_L^*}\right]=0.$$

Because $\frac{\partial F(\rho_L,\pi)}{\partial\rho_L}<0$, $\rho_L^*\gtreqless p$ holds if and only if $F(\rho_L,\pi)\big|_{\rho_L=p}\gtreqless 0$

First, $F(\rho_L,\pi)\big|_{\rho_L=p}=\tilde{u}-\frac{\pi^2(1-p)}{\pi+(1-\pi)p}$. Second, it is easy to verify that $\frac{\partial\frac{\pi^2(1-p)}{\pi+(1-\pi)p}}{\partial\pi}>0$. It then follows that $F(p,\pi)\gtreqless 0$ iff $\pi\lesseqgtr\underline{\pi}$.

What remains to be verified is $\underline{\pi}\in(0,1)$. It can be derived that $\frac{\pi^2(1-p)}{\pi+(1-\pi)p}$ approaches to 0 when π approaches to 0 and $1-p$ when π approaches to 1. Then, given that $0<\tilde{u}<1-p$, it must be true that $\underline{\pi}\in(0,1)$ if there exists a $\underline{\pi}$ such that $\tilde{u}=\frac{\underline{\pi}^2(1-p)}{\underline{\pi}+(1-\underline{\pi})p}$.

(ii) By (i), we know that $\rho_L^*\geq p>0$ when $\pi\leq\underline{\pi}$. It remains to check whether $\rho_L^*>0$ when $\pi>\underline{\pi}$.

A necessary and sufficient condition for $\rho_L^* > 0$ to hold is $F(0,\pi) > 0$. Note that $F(0,\pi) = \tilde{u} - \left[\frac{\pi(1-p)}{\pi(1-p)+(1-\pi)} - p\right]$. Since $\frac{\partial \frac{\pi(1-p)}{\pi(1-p)+(1-\pi)}}{\partial \pi} > 0$, it must be true that $F(0,\pi) > F(0,\bar{\pi}) = 0$ when $\pi < \bar{\pi}$.

Finally, it must be true that $\bar{\pi} \geq \underline{\pi}$. Suppose the opposite holds, i.e., $\bar{\pi} < \underline{\pi}$. By part (i), we know that $\rho_L^* > p > 0$ must hold when $\pi = \bar{\pi}$. But $\frac{\partial F(\rho_L,\pi)}{\partial \rho_L} < 0$ implies that $F(\rho_L^*, \bar{\pi}) < F(0,\bar{\pi}) = 0$, which contradicts with ρ_L^* being an interior equilibrium. ∎

6.5 Proof of Proposition 3

Proof. Recall that the equilibrium condition can be written

$$F(\rho_L, \tilde{u}, \pi) \;=\; 0,$$

where

$$F(\rho_L, \tilde{u}, \pi) \;\equiv\; \tilde{u} - \left[\frac{\pi(1-p)}{\pi(1-p)+(1-\pi)-(1-\pi)\rho_L^*} - \frac{\pi p}{\pi+(1-\pi)\rho_L^*}\right].$$

Let us define $A = \pi + \tilde{\rho}$ and $B = \pi(1-p) + (1-\pi) - \tilde{\rho}$. The function F can be rewritten as

$$F = \tilde{u} + \pi \frac{pB - (1-p)A}{AB}.$$

Evaluating the derivative of $F(\rho_L, \tilde{u}, \pi)$ with respect to π yields

$$\begin{aligned}
\frac{\partial F}{\partial \pi} &= \frac{\partial\left\{\pi \frac{pB-(1-p)A}{AB}\right\}}{\partial \pi} \\
&= \frac{[pB-(1-p)A]}{AB} - \pi \frac{pB^2 \frac{\partial A}{\partial \pi} - (1-p)A^2 \frac{\partial B}{\partial \pi}}{(AB)^2},
\end{aligned}$$

where $\frac{\partial A}{\partial \pi} = 1$ and $\frac{\partial B}{\partial \pi} = -p$. Hence,

$$\frac{\partial F}{\partial \pi} = \frac{[pB-(1-p)A]}{AB} - \pi p \frac{B^2 + (1-p)A^2}{(AB)^2}.$$

Noting that the existence of the interior equilibrium $\tilde{\rho}$ implies that $pB - (1-p)A < 0$, hence $\frac{\partial F}{\partial \pi} < 0$. It follows that $\frac{d\tilde{\rho}}{d\pi} = -\frac{\frac{\partial F}{\partial \pi}}{\frac{\partial F}{\partial \tilde{\rho}}} < 0$.

Using our previous result about $\frac{\partial F}{\partial \tilde{\rho}}$, we have

$$\begin{aligned}
\frac{d\tilde{\rho}}{d\pi} &= -\frac{\frac{\partial F}{\partial \pi}}{\frac{\partial F}{\partial \tilde{\rho}}} \\
&= -\frac{[(1-p)A - pB]\,AB + \pi p\left[B^2 + (1-p)A^2\right]}{\pi\left[(1-p)\,A^2 + pB^2\right]} \\
&= -\frac{[(1-p)A - pB]\,AB + \pi p\left[pB^2 + (1-p)A^2\right] + \pi p(1-p)B^2}{\pi\left[(1-p)\,A^2 + pB^2\right]} \\
&= -\frac{[(1-p)A - pB]\,AB + \pi p(1-p)B^2}{\pi\left[(1-p)\,A^2 + pB^2\right]} - p < 0.
\end{aligned}$$

Because $\tilde{\rho} = (1-\pi)\rho_L^*$, we have

$$\begin{aligned}\frac{d\rho_L^*}{d\pi} &= \frac{1}{1-\pi}[\frac{d\tilde{\rho}}{d\pi} + \rho_L^*] \\ &= \frac{1}{1-\pi}[-\frac{[(1-p)A - pB]\,AB + \pi p(1-p)B^2}{\pi\,[(1-p)\,A^2 + pB^2]} - p + \rho_L^*].\end{aligned}$$

When $\pi \geq \underline{\pi}$, $\rho_L^* \leq p$. It then follows that $\frac{d\rho_L^*}{d\pi} < 0$. Considering the possibility that $\rho_L^* = 0$ for sufficiently high π, we have $\frac{d\rho_L^*}{d\pi} \leq 0$ for $\pi \geq \underline{\pi}$. ∎

6.6 Proof of Proposition 4

Proof. (i) First, the sign of $\frac{\partial^2\tilde{\rho}}{\partial\pi\partial(e\Delta)}$ is equivalent to $\frac{\partial^2\tilde{\rho}}{\partial\pi\partial\tilde{\rho}}$ because $\frac{d\tilde{\rho}}{d(e\Delta)} > 0$.

Second, recall

$$\frac{\partial\tilde{\rho}}{\partial\pi} = -\frac{\frac{\partial F}{\partial\pi}}{\frac{\partial F}{\partial\tilde{\rho}}}.$$

Hence,

$$\frac{\partial\frac{\partial\tilde{\rho}}{\partial\pi}}{\partial\tilde{\rho}} = -\frac{\frac{\partial^2 F}{\partial\pi\partial\tilde{\rho}}\frac{\partial F}{\partial\tilde{\rho}} - \frac{\partial^2 F}{\partial^2\tilde{\rho}}\frac{\partial F}{\partial\pi}}{[\frac{\partial F}{\partial\tilde{\rho}}]^2} = \frac{\frac{\partial^2 F}{\partial^2\tilde{\rho}}\frac{\partial F}{\partial\pi} - \frac{\partial^2 F}{\partial\pi\partial\tilde{\rho}}\frac{\partial F}{\partial\tilde{\rho}}}{[\frac{\partial F}{\partial\tilde{\rho}}]^2},$$

which implies that the sign of $\partial^2\tilde{\rho}/(\partial\pi\partial\tilde{\rho})$ is determined by that of $\partial^2 F/\partial^2\tilde{\rho}\cdot\partial F/\partial\pi - \partial^2 F/(\partial\pi\partial\tilde{\rho})\cdot\partial F/\partial\tilde{\rho}$.

Recall by our previous analysis,

$$\frac{\partial F}{\partial\pi} = \frac{[pB - (1-p)A]}{AB} - \pi p\frac{B^2 + (1-p)A^2}{(AB)^2} < 0,$$

and note that $\partial A/\partial\tilde{\rho} = 1$ and $\partial B/\partial\tilde{\rho} = -1$. Evaluating the derivative of $\partial F/\partial\pi$ with respect to $\tilde{\rho}$, we have

$$\begin{aligned}\frac{\partial^2 F}{\partial\pi\partial\tilde{\rho}} &= -\frac{(1-p)\,A^2 + pB^2}{(AB)^2} - \pi p\frac{\partial\left\{\frac{1}{A^2} + \frac{(1-p)}{B^2}\right\}}{\partial\tilde{\rho}} \\ &= -\frac{(1-p)\,A^2 + pB^2}{(AB)^2} + 2\pi p\frac{B^3 - (1-p)A^3}{(AB)^3}.\end{aligned}$$

Furthermore, we have

$$\begin{aligned}\left[\frac{\partial^2 F}{\partial^2\tilde{\rho}}\frac{\partial F}{\partial\pi} - \frac{\partial^2 F}{\partial\pi\partial\tilde{\rho}}\frac{\partial F}{\partial\tilde{\rho}}\right] &= 2\pi\frac{pB^3 - (1-p)A^3}{A^3B^3}\left\{\frac{[pB - (1-p)A]}{AB} - \pi p\frac{B^2 + (1-p)A^2}{(AB)^2}\right\} \\ &\quad +\pi\left\{-\frac{(1-p)\,A^2 + pB^2}{(AB)^2} + 2\pi p\frac{B^3 - (1-p)A^3}{(AB)^3}\right\}\left\{\frac{(1-p)\,A^2 + pB^2}{(AB)^2}\right\},\end{aligned}$$

which can be rewritten as

$$
\begin{aligned}
& \left[\frac{\partial^2 F}{\partial^2 \tilde{\rho}} \frac{\partial F}{\partial \pi} - \frac{\partial^2 F}{\partial \pi \partial \tilde{\rho}} \frac{\partial F}{\partial \tilde{\rho}}\right] \Big/ \pi \\
= \;& 2\frac{pB^3 - (1-p)A^3}{(AB)^3} \left\{ \frac{[pB - (1-p)A]}{AB} - \pi p \frac{B^2 + (1-p)A^2}{(AB)^2} \right\} \\
& + \left\{ -\frac{(1-p)\,A^2 + pB^2}{(AB)^2} + 2\pi p \frac{B^3 - (1-p)A^3}{(AB)^3} \right\} \left\{ \frac{(1-p)\,A^2 + pB^2}{(AB)^2} \right\} \\
= \;& \frac{[(1-p)A^3 - pB^3]\,[(1-p)A - pB] + 2\pi p(1-p)^2 AB(A+B) - p(1-p)\,[A^3B + AB^3 + 2A^2B^2]}{A^4B^4} \\
= \;& \frac{[(1-p)A^3 - pB^3]\,[(1-p)A - pB] - p(1-p)AB(A+B)[1 - \pi(1-p)]}{A^4B^4}.
\end{aligned}
$$

The above means that the sign of $\frac{\partial^2 \tilde{\rho}}{\partial \pi \partial \tilde{\rho}}$ is equivalent to the sign of the numerator, $[(1-p)A^3 - pB^3]\,[(1-p)A - pB] - p(1-p)AB(A+B)[1-\pi(1-p)]$. Note that an equilibrium requires $\tilde{u} = \pi \frac{[(1-p)A - pB]}{AB}$. Hence, the sign of the numerator is further the same as that of

$$
\begin{aligned}
G(\pi, \tilde{\rho}(\pi)) &= \left[(1-p)A^3 - pB^3\right] \tilde{u} - p(1-p)\pi(A+B)[1 - \pi(1-p)] \\
&= \left[(1-p)A^3 - pB^3\right] \tilde{u} - p(1-p)\pi\,[1 + \pi(1-p)]\,[1 - \pi(1-p)].
\end{aligned}
$$

$G(\pi, \tilde{\rho}(\pi))$ must be positive when π is close to zero. First, $p(1-p)\pi\,[1+\pi(1-p)]\,[1 - \pi(1-p)]$ approaches to zero. Second, $[(1-p)A^3 - pB^3]\,\tilde{u}$ is strictly positive. This is because that an interior equilibrium implies $pB - (1-p)A < 0$, which further implies that $\frac{B}{A} < \frac{1-p}{p}$. Together with $p \geq \frac{1}{2}$, it immediately follows that $\frac{B}{A} < 1$. It then implies that $\frac{B^3}{A^3} < \frac{B}{A} < \frac{1-p}{p}$, i.e., $(1-p)A^3 - pB^3 > 0$.

We then verify that $\frac{\partial^2 \tilde{\rho}}{\partial \pi \partial (e\Delta)}$ must be strictly positive for small π.

(ii) We first explore the property of the function $G(\pi, \tilde{\rho}(\pi))$. Recall $(1-p)A^3 - pB^3 = (1-p)(\pi + \tilde{\rho})^3 - p\,[\pi(1-p) + (1-\pi) - \tilde{\rho}]^3$. It is obvious to see that $G_{\tilde{\rho}}(\pi, \tilde{\rho})$ is strictly positive for $\tilde{\rho} \in [0,1]$. We now consider $\frac{dG(\pi, \tilde{\rho}(\pi))}{d\pi}$. First, evaluating the derivative of $\pi\,[1 + \pi(1-p)]\,[1 - \pi(1-p)]$ with respect to π gives $1 - 3\pi^2(1-p)^2$, which is positive for any $p \geq \frac{1}{2}$. Second, $\frac{d\left[(1-p)A^3 - pB^3\right]}{d\pi} = 3\,[(1-p)A^2 + p^2B^2] + 3\,[(1-p)A^2 + pB^2]\,\frac{d\tilde{\rho}}{d\pi}$. By our previous analysis,

$$
\frac{d\tilde{\rho}}{d\pi} = -\frac{[(1-p)A - pB]\,AB + \pi p\,[B^2 + (1-p)A^2]}{\pi\,[(1-p)\,A^2 + pB^2]}.
$$

Hence,

$$
\begin{aligned}
\frac{d\,[(1-p)A^3 - pB^3]}{d\pi} &= 3\left[(1-p)A^2 + p^2B^2\right] \\
&\quad -3\left[(1-p)A^2 + pB^2\right] \frac{[(1-p)A - pB]\,AB + \pi p\,[B^2 + (1-p)A^2]}{\pi\,[(1-p)\,A^2 + pB^2]} \\
&= \frac{3}{\pi}\left\{\pi(1-p)\left[(1-p)A^2 - pB^2\right] - [(1-p)A - pB]\,AB\right\},
\end{aligned}
$$

and,

$$
\begin{aligned}
\frac{dG(\pi,\tilde{\rho}(\pi))}{d\pi}\pi &= \pi\frac{d\left[(1-p)A^3-pB^3\right]}{d\pi}\tilde{u}-\pi p(1-p)[1-3\pi^2(1-p)^2] \\
&= 3\left\{\pi(1-p)\left[(1-p)A^2-pB^2\right]-\left[(1-p)A-pB\right]AB\right\}\tilde{u} \\
&\quad -\pi p(1-p)\left[1-3\pi^2(1-p)^2\right] \\
&= 3\left\{-(1-p)A^2[(1-\pi)-\tilde{\rho}]+pB^2(\pi p+\tilde{\rho})\right\}\tilde{u} \\
&\quad -\pi p(1-p)\left[1-3\pi^2(1-p)^2\right] \\
&= 3\left\{-(1-p)A^2[(1-\pi)-\tilde{\rho}]+pB^2(\pi p+\tilde{\rho})\right\}\frac{\pi\left[(1-p)A-pB\right]}{AB} \\
&\quad -\pi p(1-p)\left[1-3\pi^2(1-p)^2\right] \\
&= \frac{\pi}{AB}\left\{\begin{array}{c} 3\left\{-(1-p)A^2[(1-\pi)-\tilde{\rho}]+pB^2(\pi p+\tilde{\rho})\right\}\left[(1-p)A-pB\right] \\ -ABp(1-p)\left[1-3\pi^2(1-p)^2\right]\end{array}\right\}.
\end{aligned}
$$

It is easy to verify that $(1-p)A-pB=\pi(1-p+p^2)+\tilde{\rho}-p<\pi(1-p)^2$ for $\pi\geq\underline{\pi}$ because $\tilde{\rho}<\rho_L\leq p$. As a result,

$$
\begin{aligned}
\frac{dG(\pi,\tilde{\rho}(\pi))}{d\pi}\pi &< \frac{\pi(1-p)}{AB}\left\{\begin{array}{c} 3\left\{-(1-p)A^2[(1-\pi)-\tilde{\rho}]+pB^2(p\pi+\tilde{\rho})\right\}(1-p) \\ -ABp\left[1-3\pi^2(1-p)^2\right]\end{array}\right\} \\
&= \frac{\pi(1-p)}{AB}\left\{\begin{array}{c} 3\left\{\begin{array}{c}-(1-p)A^2[(1-\pi)-\tilde{\rho}]+pB^2(p\pi+\tilde{\rho}) \\ +pAB\pi^2(1-p)\end{array}\right\}(1-p) \\ -ABp\end{array}\right\}
\end{aligned}
$$

For $\pi\geq\underline{\pi}$, $p\pi+\tilde{\rho}\leq p\pi+(1-\pi)p=p$. Hence,

$$
\begin{aligned}
&\left\{\begin{array}{c} 3\left\{\begin{array}{c}-(1-p)A^2[(1-\pi)-\tilde{\rho}]+pB^2(p\pi+\tilde{\rho}) \\ +pAB\pi^2(1-p)\end{array}\right\}(1-p) \\ -ABp\end{array}\right\} \\
\leq &\left\{\begin{array}{c} 3\left\{\begin{array}{c}-(1-p)A^2[(1-\pi)-\tilde{\rho}]+p^2B^2 \\ +pAB\pi^2(1-p)\end{array}\right\}(1-p) \\ -ABp\end{array}\right\}.
\end{aligned}
$$

Because $(1-p)A>pB$,

$$
\begin{aligned}
&\left\{\begin{array}{c} 3\left\{\begin{array}{c}-(1-p)A^2[(1-\pi)-\tilde{\rho}]+p^2B^2 \\ +pAB\pi^2(1-p)\end{array}\right\}(1-p) \\ -ABp\end{array}\right\} \\
< &\left\{\begin{array}{c} 3\left\{\begin{array}{c}-pAB[(1-\pi)-\tilde{\rho}]+p(1-p)AB \\ +pAB\pi^2(1-p)\end{array}\right\}(1-p) \\ -ABp\end{array}\right\}.
\end{aligned}
$$

For our purpose, we only need to identify the condition for

$$3\left\{-[(1-\pi)-\tilde{\rho}]+(1-p)+\pi^2(1-p)\right\}(1-p)<1.$$

Note that for $\pi \geq \underline{\pi}$, $(1-\pi)-\tilde{\rho}=(1-\pi)-(1-\pi)\rho_L \geq (1-\pi)(1-p)$. Hence,

$$\begin{aligned}
& 3\left\{-[(1-\pi)-\tilde{\rho}]+(1-p)+\pi^2(1-p)\right\}(1-p) \\
\leq\ & 3\left\{\pi(1-p)+\pi^2(1-p)\right\}(1-p) \\
=\ & 3\pi(1-p)^2(1+\pi).
\end{aligned}$$

Because $\pi(1+\pi)<2$, a sufficient condition would be $(1-p)^2 \leq \frac{1}{6} \Leftrightarrow p \geq 1-\frac{1}{\sqrt{6}} \approx 0.59$.

We now verify that for $\pi=\underline{\pi}$, $G(\pi,\tilde{\rho})<0$. To see that, recall

$$G(\pi,\tilde{\rho})=\left[(1-p)A^3-pB^3\right]\tilde{u}-p(1-p)\pi\left[1+\pi(1-p)\right]\left[1-\pi(1-p)\right].$$

When $\pi=\underline{\pi}$, $\rho_L=p$ and $B=1-p$. Hence,

$$\begin{aligned}
G(\pi,\tilde{\rho}) &= \left[(1-p)A^3-pB^3\right]\tilde{u}-p(1-p)\underline{\pi}\left[1+\underline{\pi}(1-p)\right]\left[1-\underline{\pi}(1-p)\right] \\
&= (1-p)\left[A^3-p(1-p)^2\right]\tilde{u}-p(1-p)\underline{\pi}\left[1+\underline{\pi}(1-p)\right]\left[1-\underline{\pi}(1-p)\right] \\
&= (1-p)\left\{\left[A^3-p(1-p)^2\right]\tilde{u}-p\underline{\pi}\left[1+\underline{\pi}(1-p)\right]\left[1-\underline{\pi}(1-p)\right]\right\}.
\end{aligned}$$

Further, for $\pi=\underline{\pi}$, $\tilde{u}=\frac{\pi^2(1-p)}{\pi+(1-\pi)p}$. Hence,

$$\begin{aligned}
G(\pi,\tilde{\rho}) &= (1-p)\left\{\left[A^3-p(1-p)^2\right]\frac{\underline{\pi}^2(1-p)}{\underline{\pi}+(1-\underline{\pi})p}-p\underline{\pi}\left[1+\underline{\pi}(1-p)\right]\left[1-\underline{\pi}(1-p)\right]\right\} \\
&= \underline{\pi}(1-p)\left\{\left[A^3-p(1-p)^2\right]\frac{\underline{\pi}(1-p)}{\underline{\pi}+(1-\underline{\pi})p}-p\left[1+\underline{\pi}(1-p)\right]\left[1-\underline{\pi}(1-p)\right]\right\}.
\end{aligned}$$

Because $\underline{\pi}+(1-\underline{\pi})p=A$,

$$\begin{aligned}
G(\pi,\tilde{\rho}) &= \underline{\pi}(1-p)\left\{\left[A^3-p(1-p)^2\right]\frac{\underline{\pi}(1-p)}{A}-p\left[1-\underline{\pi}^2(1-p)^2\right]\right\} \\
&= \underline{\pi}(1-p)\left\{\frac{\left[A^3-p(1-p)^2+Ap\underline{\pi}(1-p)\right]}{A}\underline{\pi}(1-p)-p\right\} \\
&= \underline{\pi}(1-p)\left\{\frac{\left[(A-p)A^2+pA^2-pB^2+Ap\underline{\pi}(1-p)\right]}{A}\underline{\pi}(1-p)-p\right\}
\end{aligned}$$

Recall that at $\pi=\underline{\pi}$, $A=\underline{\pi}+(1-\underline{\pi})\,p$, which gives $A-p=\underline{\pi}+(1-\underline{\pi})\,p-p=\underline{\pi}(1-p)$.

Hence,

$$
\begin{aligned}
G(\pi,\tilde{\rho}) &= \underline{\pi}(1-p)\left\{\frac{\left[\underline{\pi}(1-p)A^2+p\left(A^2-B^2\right)+Ap\underline{\pi}(1-p)\right]}{A}\underline{\pi}(1-p)-p\right\} \\
&= \underline{\pi}(1-p)\left\{\frac{\left[\underline{\pi}(1-p)A^2+p(A+B)(A-B)+Ap\underline{\pi}(1-p)\right]}{A}\underline{\pi}(1-p)-p\right\} \\
&= \underline{\pi}(1-p)\left\{\frac{\left[\begin{array}{c}\underline{\pi}(1-p)A^2+p(A+B)\left[p-(1-\underline{\pi})(1-p)\right]\\ +A\underline{\pi}(1-p)\end{array}\right]}{A}\underline{\pi}(1-p)-p\right\} \\
&= \underline{\pi}(1-p)\left\{\left\{\begin{array}{c}\underline{\pi}(1-p)A+\underline{\pi}p(1-p)+\frac{p^2(A+B)}{A}\\ -\frac{p(A+B)(1-\underline{\pi})(1-p)}{A}\end{array}\right\}\underline{\pi}(1-p)-p\right\} \\
&= \underline{\pi}p(1-p)\left\{\left\{\begin{array}{c}\frac{\underline{\pi}(1-p)\left[\underline{\pi}+(1-\underline{\pi})p\right]}{p}+\underline{\pi}(1-p)\\ +p+\frac{pB}{A}-\frac{(A+B)(1-\underline{\pi})(1-p)}{A}\end{array}\right\}\underline{\pi}(1-p)-1\right\} \\
&= \underline{\pi}p(1-p)\left\{\left\{\begin{array}{c}\frac{\underline{\pi}^2(1-p)}{p}+\underline{\pi}(1-\underline{\pi})(1-p)+\underline{\pi}+p(1-\underline{\pi})\\ +\frac{pB}{A}-\frac{(1-\underline{\pi})(1-p)A}{A}-\frac{B(1-\underline{\pi})(1-p)}{A}\end{array}\right\}\underline{\pi}(1-p)-1\right\}.
\end{aligned}
$$

Recall that in equilibrium $pB<(1-p)A$, hence $\frac{pB}{A}-\frac{(1-\underline{\pi})(1-p)A}{A}<\underline{\pi}(1-p)$, and

$$
G(\pi,\tilde{\rho})<\underline{\pi}p(1-p)\left\{\left\{\frac{\underline{\pi}^2(1-p)}{p}+\underline{\pi}(1-\underline{\pi})(1-p)+\underline{\pi}+p(1-\underline{\pi})+\underline{\pi}(1-p)-\frac{B(1-\underline{\pi})(1-p)}{A}\right\}\underline{\pi}(1-p
$$

Further,

$$
\begin{aligned}
&\left\{\frac{\underline{\pi}^2(1-p)}{p}+\underline{\pi}(1-\underline{\pi})(1-p)+\underline{\pi}+p(1-\underline{\pi})+\underline{\pi}(1-p)-\frac{B(1-\underline{\pi})(1-p)}{A}\right\}\underline{\pi}(1-p) \\
<&\left\{\frac{\underline{\pi}^2(1-p)}{p}+\underline{\pi}(1-\underline{\pi})(1-p)+\underline{\pi}+p(1-\underline{\pi})+\underline{\pi}(1-p)\right\}\underline{\pi}(1-p) \\
=&\ \frac{\underline{\pi}^3(1-p)^2}{p}+\underline{\pi}^2(1-\underline{\pi})(1-p)^2+\underline{\pi}\left[\underline{\pi}(1-p)+(1-\underline{\pi})p\right](1-p)+\underline{\pi}^2(1-p),
\end{aligned}
$$

which, we claim, is strictly less than one. First, given that $p\geq 1-\frac{1}{\sqrt{6}}\approx 0.59$, $\frac{\underline{\pi}^3(1-p)^2}{p}<0.29$. Second, consider $\underline{\pi}^2(1-\underline{\pi})(1-p)^2$. The function $\underline{\pi}^2(1-\underline{\pi})$ is maximized when $\underline{\pi}=\frac{2}{3}$. Hence, $\underline{\pi}^2(1-\underline{\pi})(1-p)^2$ must be less than $\frac{4}{27\times 6}=0.025$. Third, $\underline{\pi}^2(1-p)$ is no more than 0.41. Finally, we consider $\underline{\pi}\left[\underline{\pi}(1-p)+(1-\underline{\pi})p\right](1-p)$. The inequality is verified as long as the $\underline{\pi}\left[\underline{\pi}(1-p)+(1-\underline{\pi})p\right](1-p)$ is less than 0.275. Evaluate the function $\underline{\pi}\left[\underline{\pi}(1-p)+(1-\underline{\pi})p\right]$ with respect to $\underline{\pi}$, which gives

$$
\begin{aligned}
&\left[\underline{\pi}(1-p)+(1-\underline{\pi})p\right]+\underline{\pi}(1-2p) \\
=&\ 2\underline{\pi}(1-2p)+p.
\end{aligned}
$$

Its maximizer is $\underline{\pi}^* = 1$ if $p < \frac{2}{3}$, or $\underline{\pi}^* = \frac{p}{2(2p-1)}$ if $p \geq \frac{2}{3}$. If $p < \frac{2}{3}$, the function's maximal value is $\underline{\pi}\left[\underline{\pi}(1-p) + (1-\underline{\pi})p\right]\big|_{\underline{\pi}=1} = (1-p)$, so $\underline{\pi}\left[\underline{\pi}(1-p) + (1-\underline{\pi})p\right]\big|_{\underline{\pi}=1}(1-p) = (1-p)^2 \in (\frac{1}{9}, \frac{1}{4}]$, which satisfies the requirement. If $p \geq \frac{2}{3}$, then the function has a maximal value

$$\begin{aligned}\underline{\pi}\left[\underline{\pi}(1-p) + (1-\underline{\pi})p\right]\big|_{\underline{\pi}=\frac{p}{2(2p-1)}} &= \frac{p}{2(1-2p)}\left[\frac{p(1-p)}{2(2p-1)} + \frac{p(3p-2)}{2(2p-1)}\right] \\ &= \frac{p^2}{4(2p-1)}.\end{aligned}$$

The function $\underline{\pi}\left[\underline{\pi}(1-p) + (1-\underline{\pi})p\right](1-p)$ thus has a maximal value $\frac{p^2(1-p)}{4(2p-1)}$. Evaluating it with respect to p obtains

$$\begin{aligned}\frac{p^2(1-p)}{4(2p-1)} &= \frac{p(2-3p)(2p-1) - 2p^2(1-p)}{4(2p-1)^2} \\ &= \frac{p}{4(2p-1)^2}\left[(2-3p)(2p-1) - 2p(1-p)\right] \\ &= -\frac{p}{4(2p-1)^2}(4p^2 - 5p + 2) \\ &= -\frac{p}{4(2p-1)^2}\left[4(p-\frac{5}{8})^2 + \frac{7}{16}\right] < 0.\end{aligned}$$

Hence, when $p \geq \frac{2}{3}$, the value of this expression is capped by

$$\frac{p^2(1-p)}{4(2p-1)}\bigg|_{p=\frac{2}{3}} = \frac{\frac{4}{27}}{\frac{4}{3}} = \frac{1}{9},$$

which is strictly less than 0.275.

We then verify that at $\pi = \underline{\pi}$, the function $G(\pi, \tilde{\rho}) < 0$. It implies that the function is strictly negative for all $\pi \in (\underline{\pi}, \bar{\pi})$. ∎

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